Shiraz University

Fetal Cardiac Signal Processing Techniques

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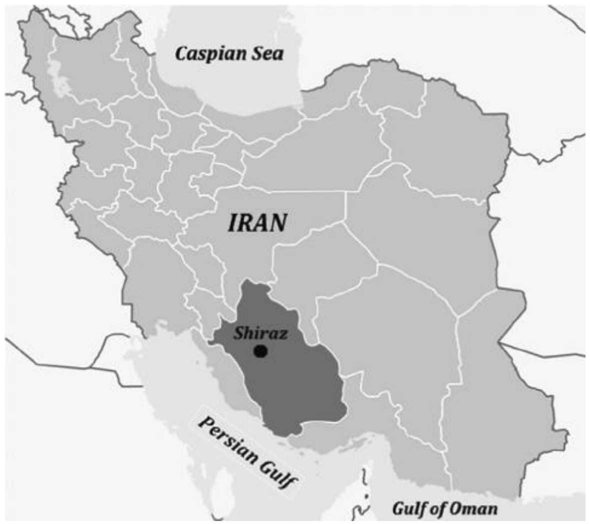
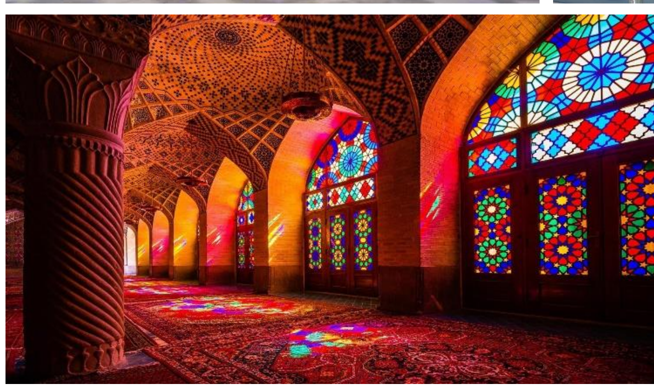
Where I come from…





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PART I

Background

Electrophysiology of the Fetal Heart

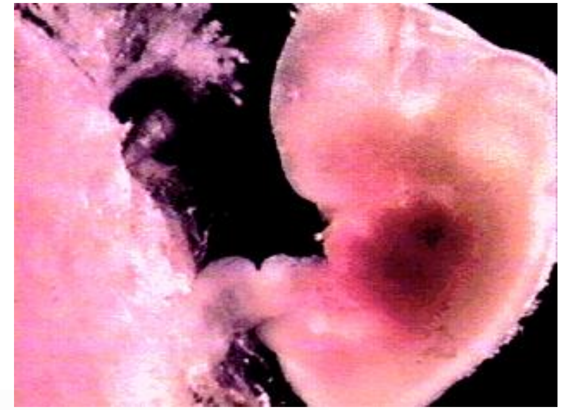
A Short Introduction

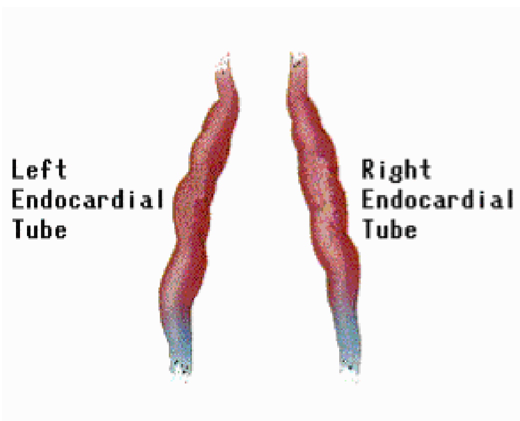
Introduction

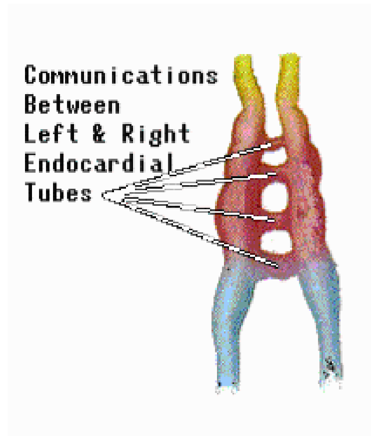
- One out of 125 babies are born with congenital heart defects
- Early detection of cardiac abnormalities help medications and precautions during pregnancy and delivery
- Most cardiac defects manifest in the heart-rate and morphology of the electrocardiogram (ECG) and the magnetocardiogram (MCG)
- But, we don't have direct access to the fetus and the fetal signals recorded from the mother's abdomen are very weak with high interferences
- Signal processing has been used to retrieve and analyze these signals over the past 50 years

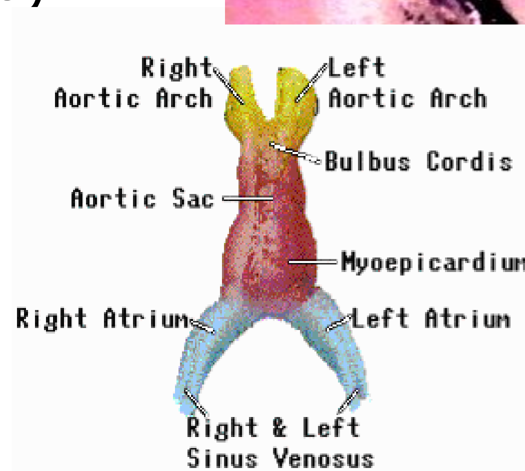
The Fetal Heart

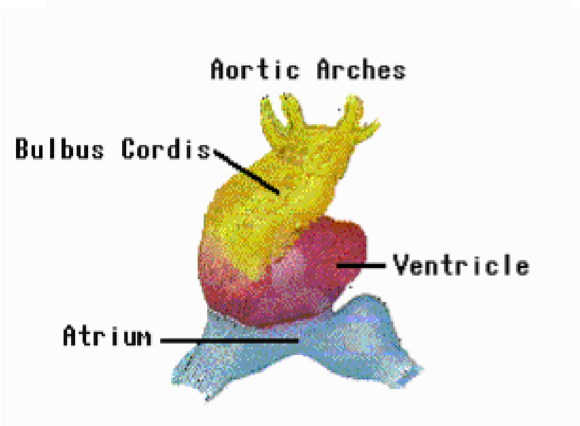
- The myocardium is among the first fetal organs that are developed during pregnancy

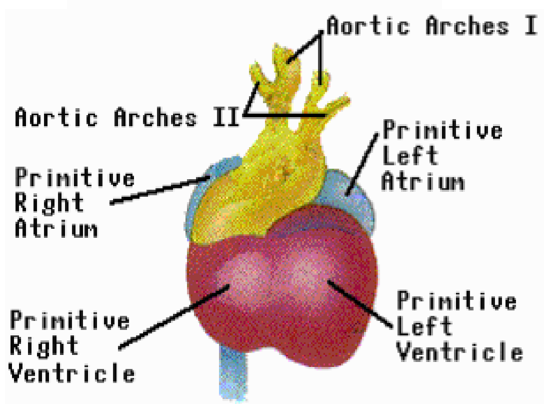


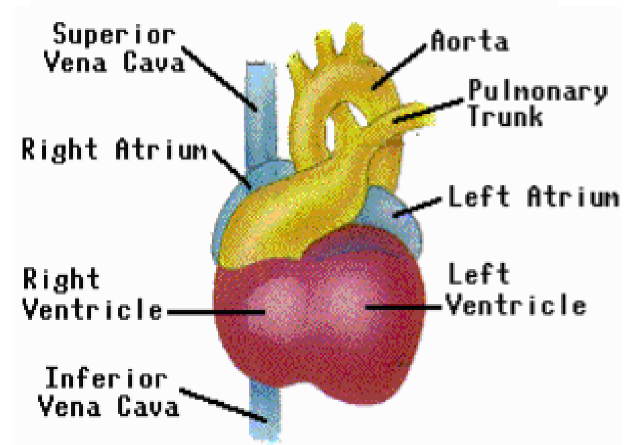








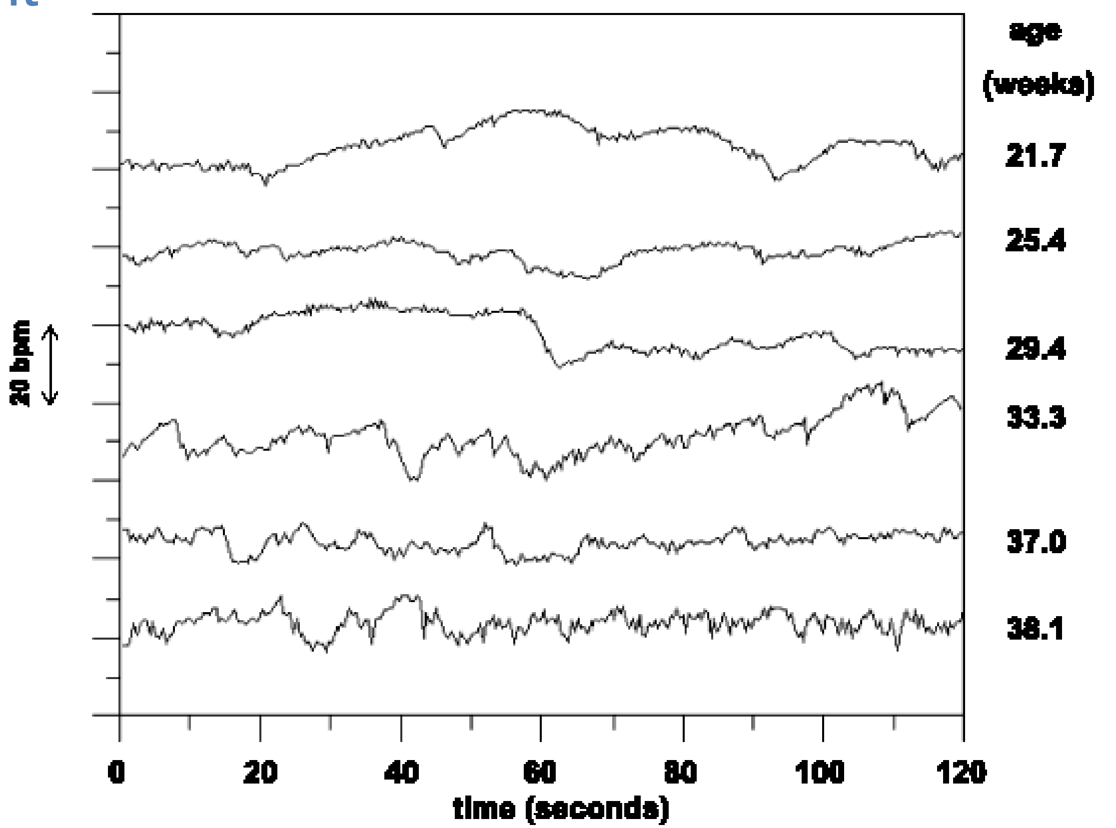




Adopted with permission from: C. D. Lawrence, Fetal Heart Development, 1995.

Evolution of Fetal Cardiac Rhythms throughout Pregnancy

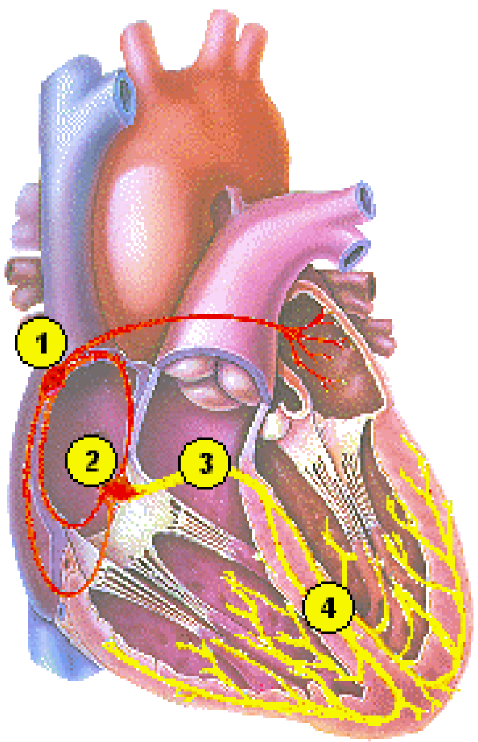
- The heart rate rhythm becomes more complex with fetal gestation age → can be used as an indicator of fetal brain and neural development



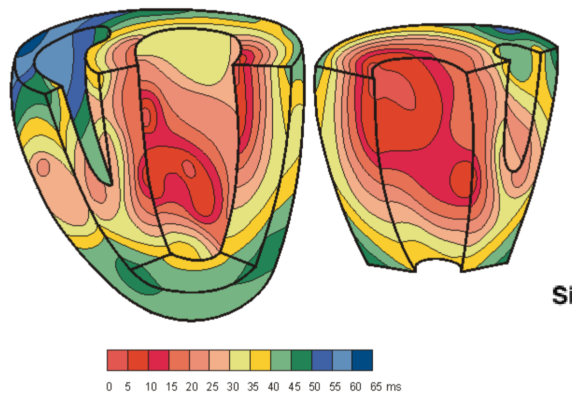
R. T. Wakai, Assessment of fetal neurodevelopment via fetal magnetocardiography. Exp Neurol, vol. 190 Suppl 1, pp. S65-S71, Nov 2004.

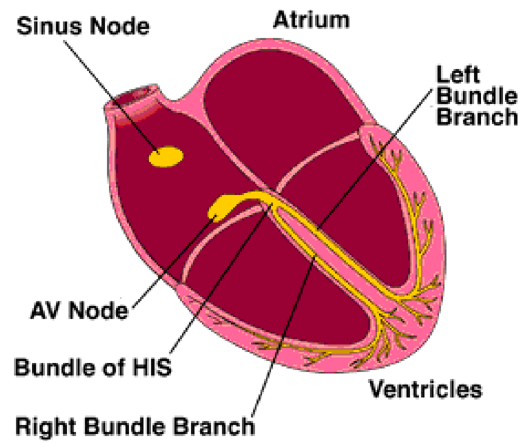
Electrical Activity of the Heart

- Cardiac contraction is initiated by its autonomous nervous system.



The heart as a muscle

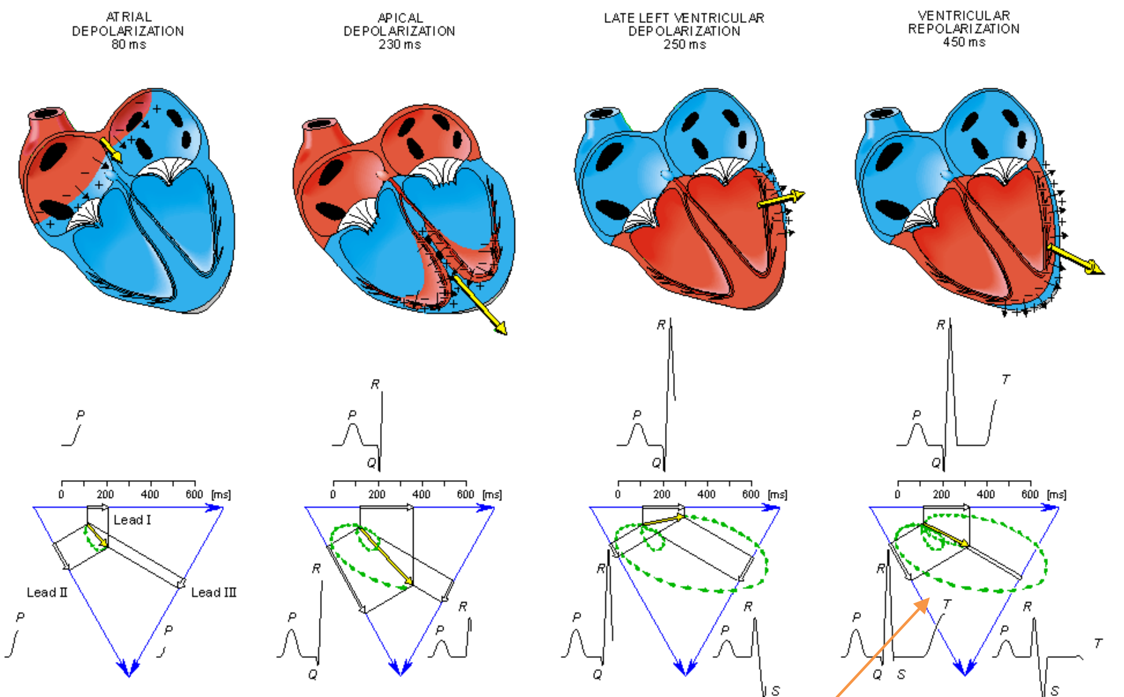




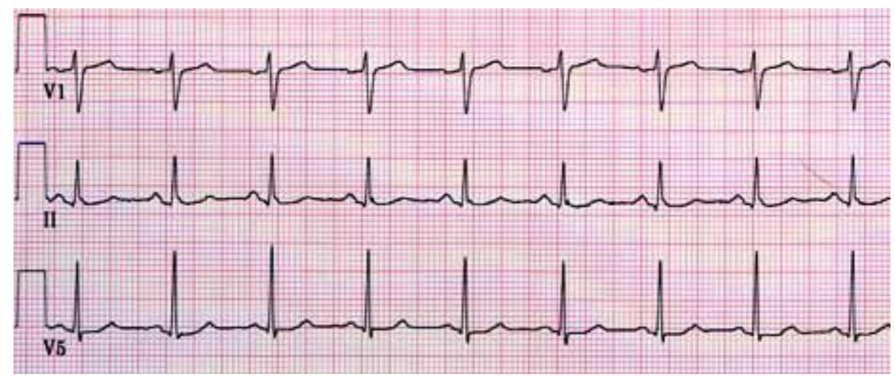
The heart as a surface of potentials

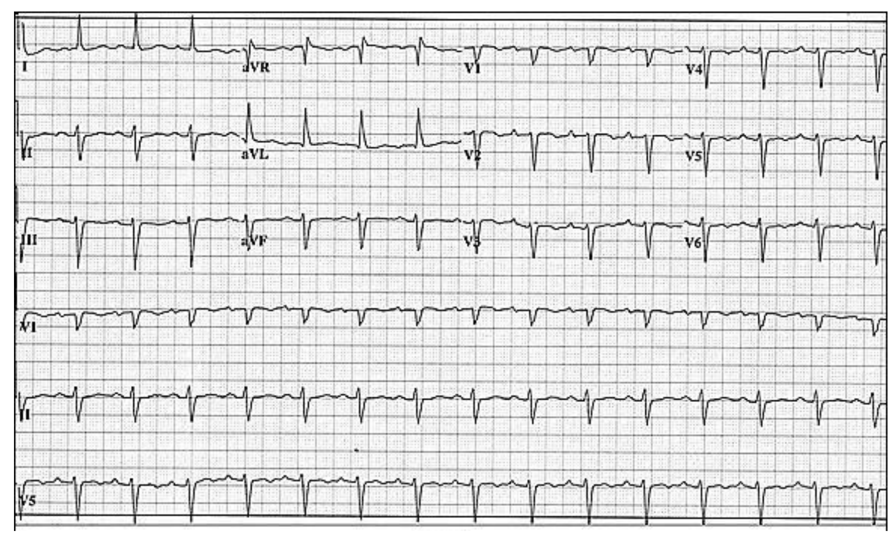
The Electrocardiogram

- The electrocardiogram (ECG) is the overall electrical activity of the heart, recorded from the body surface
- The ECG shape depends on the lead location



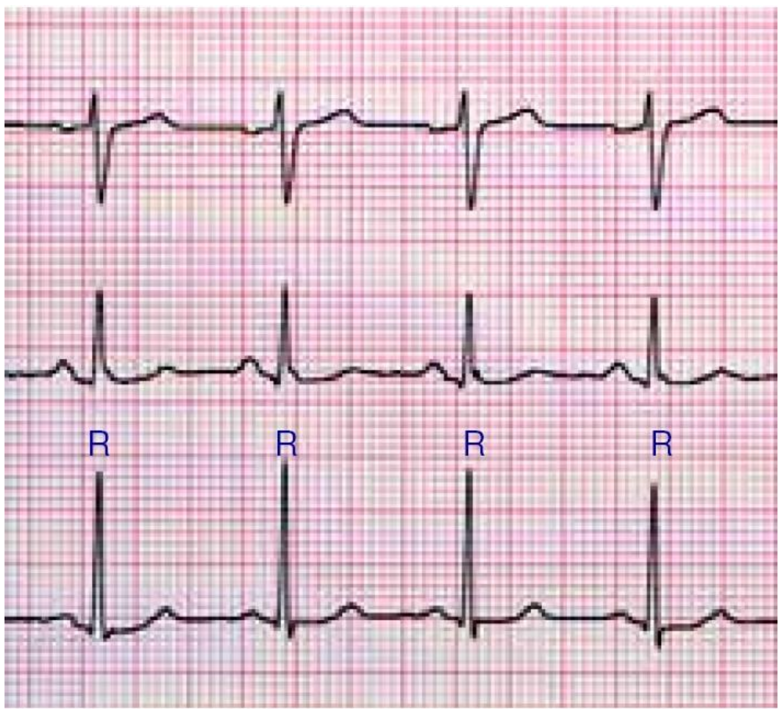
The vectorcardiogram (VCG): a 3D phase-representation of three ECG channels





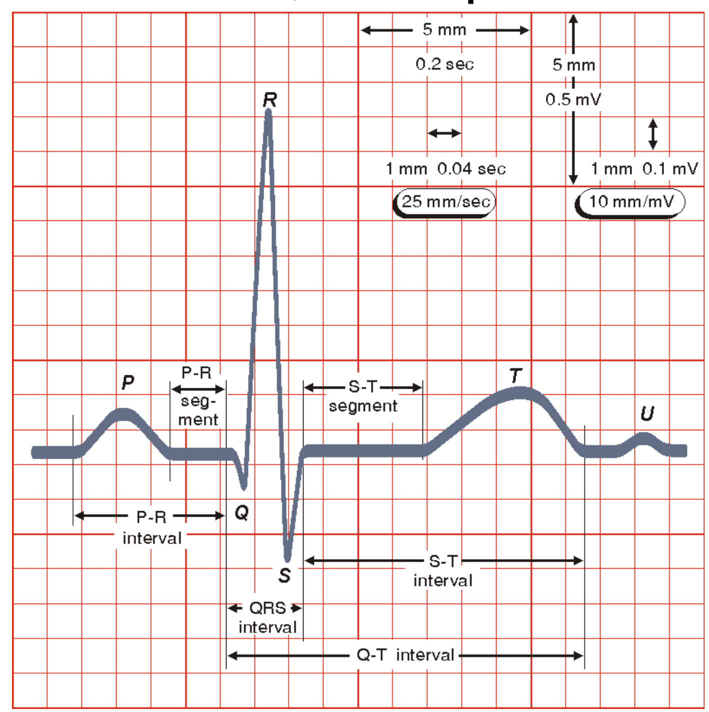
Important ECG Factors

- The major ECG information are embedded in its RR-interval time-series and beat-wise morphology
- The same factors are important for the fetal ECG



RR-Intervals (Heart Rate)

The PQRST Complex



Morphology

Signal Processing Aspects of the Heart

- A distributed source, which can only be approximated by a dipole in the “far-field”

  → Multichannel acquisition and analysis is required

- A random electrical field with cyclostationary behavior, due to the SA-node pacemaker cells

  → Knowledge of stochastic and cyclostationary processes is required

- Occasional biphasic and multiphasic (or even chaotic) behavior are observable (bigeminy, trigeminy, etc.)

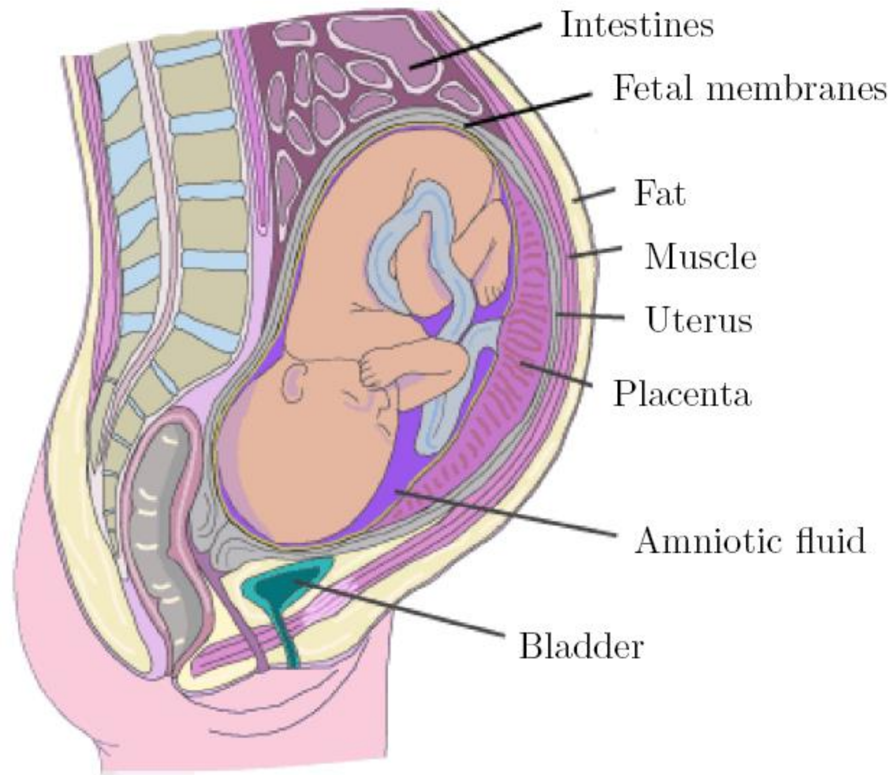
  → Knowledge of nonlinear dynamics & chaos is required

- A mixture of discrete-time and continuous-time dynamical system, due to heart-rate and morphologic coupling

  → Knowledge of Markov chains and continuous-time (cyclostationary) dynamic systems is required

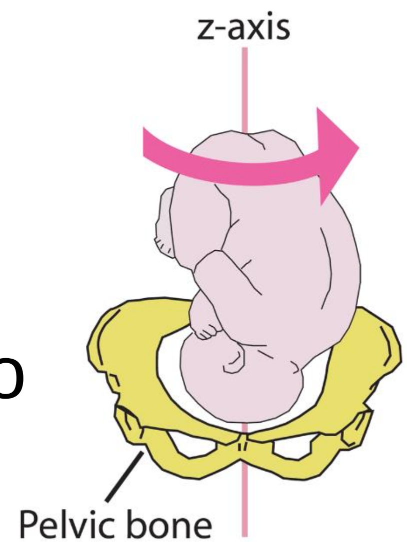
Fetal Heart Signal Processing Issues

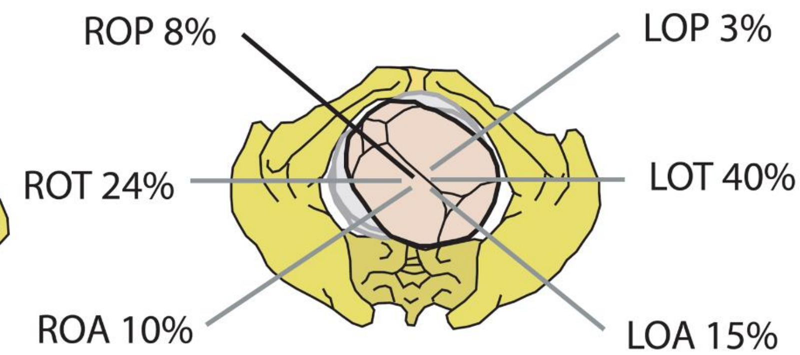
- Weakness of fetal cardiac signals
- Strong maternal ECG and respiration interference
- Movements of the fetus and electrode positioning
- Multiple pregnancies (twin, tripling, ...)
- Electrically insulating layers such as the *vernix caseosa*

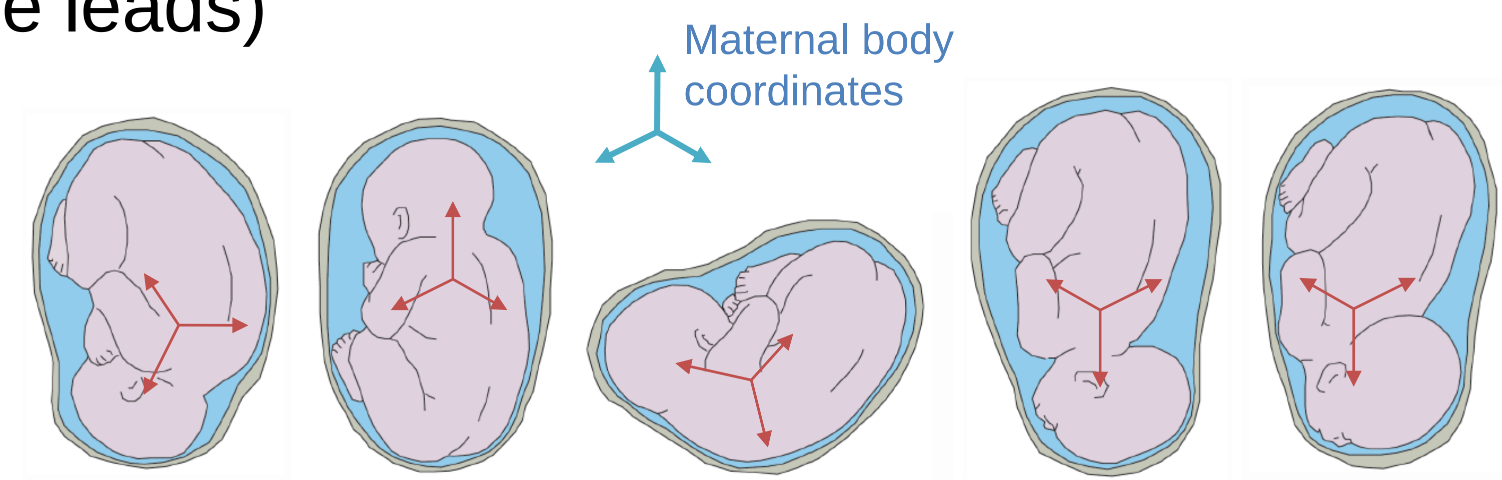


Various Fetal Positions

- The fetal body coordinates varies with fetal movements and rotations with respect to the maternal body coordinates (and the surface leads)







(a) Vertex 96.8 % (b) Breech 2.5 % (c) Shoulder 0.4 % (d) Face 0.2 % (e) Brow 0.1 %

Fetal Electrocardiography

Electrical Properties, Methods, Devices, Interferences/Noise

Common Fetal Cardiac Monitoring Techniques

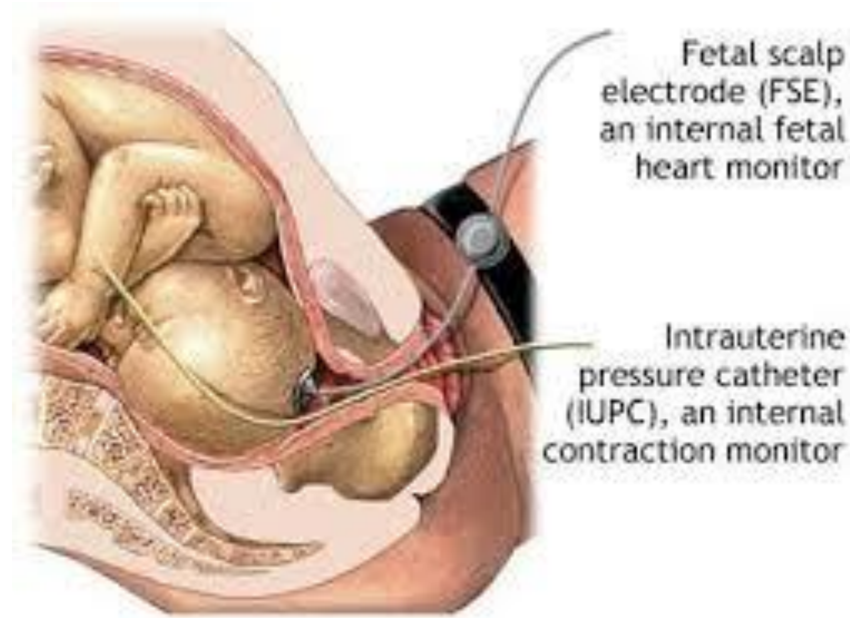
- Echocardiography: sonograghy of the heart using Doppler ultrasound [Wladimiroff & Pilu, 1996, Drose, 1998]
- Phonocardiography: heart sounds using acoustic microphones [Zuckerwar et al., 1993, Varady et al., 2003, Samieinasab 2015]
- Magnetocardiography: magnetic fields of cardiac signals [Kariniemi & Hukkinen, 1977, Stinstra, 2001]
- Electrocardiography: electric fields of cardiac signals
  - **Invasive:** measurable during labor only [Outram et al., 1995, Lai & Shynk, 2002]
  - **Noninvasive:** measurable throughout pregnancy [Cremer, 1906, Lindsley, 1942]

Fetal ECG Recording Methods

- The fetal ECG (fECG) is a balance between price and accuracy
- The two possible methods of fECG acquisition are:
  - Invasive (Direct) Methods: Fetal scalp electrode
  - Noninvasive (Indirect) Methods: Maternal abdominal leads

Invasive Fetal ECG Recording

- Used in normal vaginal deliveries after amniotic sac (fluid) rupture
- Used to identify fetuses at risk of hypoxia (oxygen deficiency) during labor



Drawbacks:

1. Only applicable during labor
2. May result in scalp lesions and newborn infections
3. May result in maternal infections

Noninvasive Fetal ECG Recording

- Can be used throughout pregnancy (especially during the third trimester) without any risks

Drawbacks: Challenges of noninvasive fetal ECG extraction

The Physics of Noninvasive Fetal ECG

The volume conductor between the fetal heart and the maternal body surface has the following properties:

1. The electric displacement current is negligible: the electromagnetic problem is quasi-static (the electric and magnetic fields are decoupled, the electric field is proportional to the gradient of the electric scalar potential and the divergence of the current density is zero).
2. The problem is linear: superposition holds for the electrical potentials due to several sources (mother; fetuses; etc.).
3. The capacitive component of the electrical impedance of body tissues is negligible (due to low frequency): the tissues are with a very good approximation resistive.

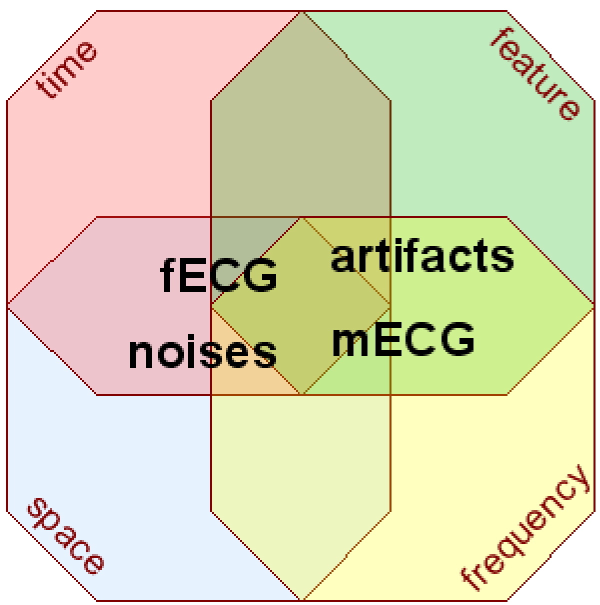
Ref: D. B. Geselowitz, On the Theory of the Electrocardiogram," Proc. IEEE, vol. 77, pp. 857-876, Jun. 1989.

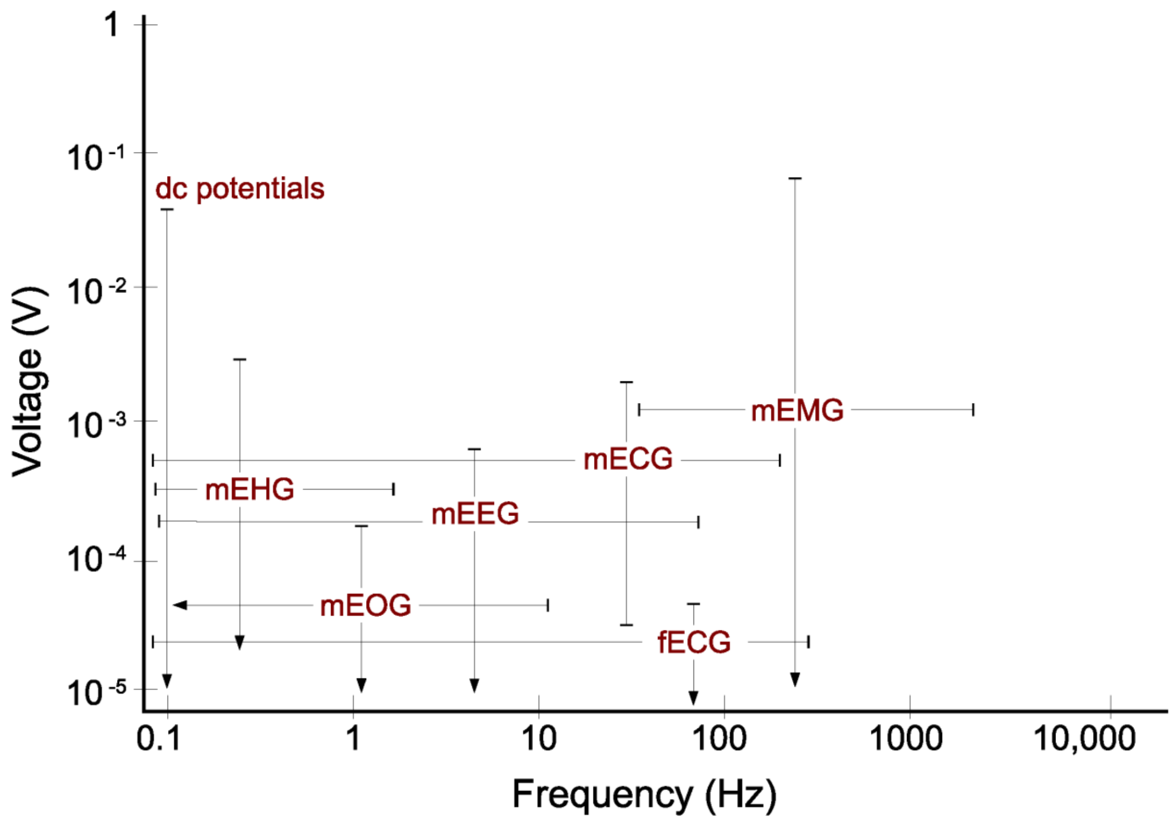
Major Fetal ECG Noises and Interferences

- The Maternal ECG (the major interference)
- Other fetal interferences (in multiple pregnancies)
- Maternal uterus and muscle contractions
- Baseline wander due to maternal respiration
- Device noise (inductive and conductive)
- Power-line noise
- Plus, the intrinsic fetal ECG issues: weakness of the fECG, morphological changes due to fetal movements, indirect access through volume conductor, etc.

Major Fetal ECG Noises and Interferences (continued)

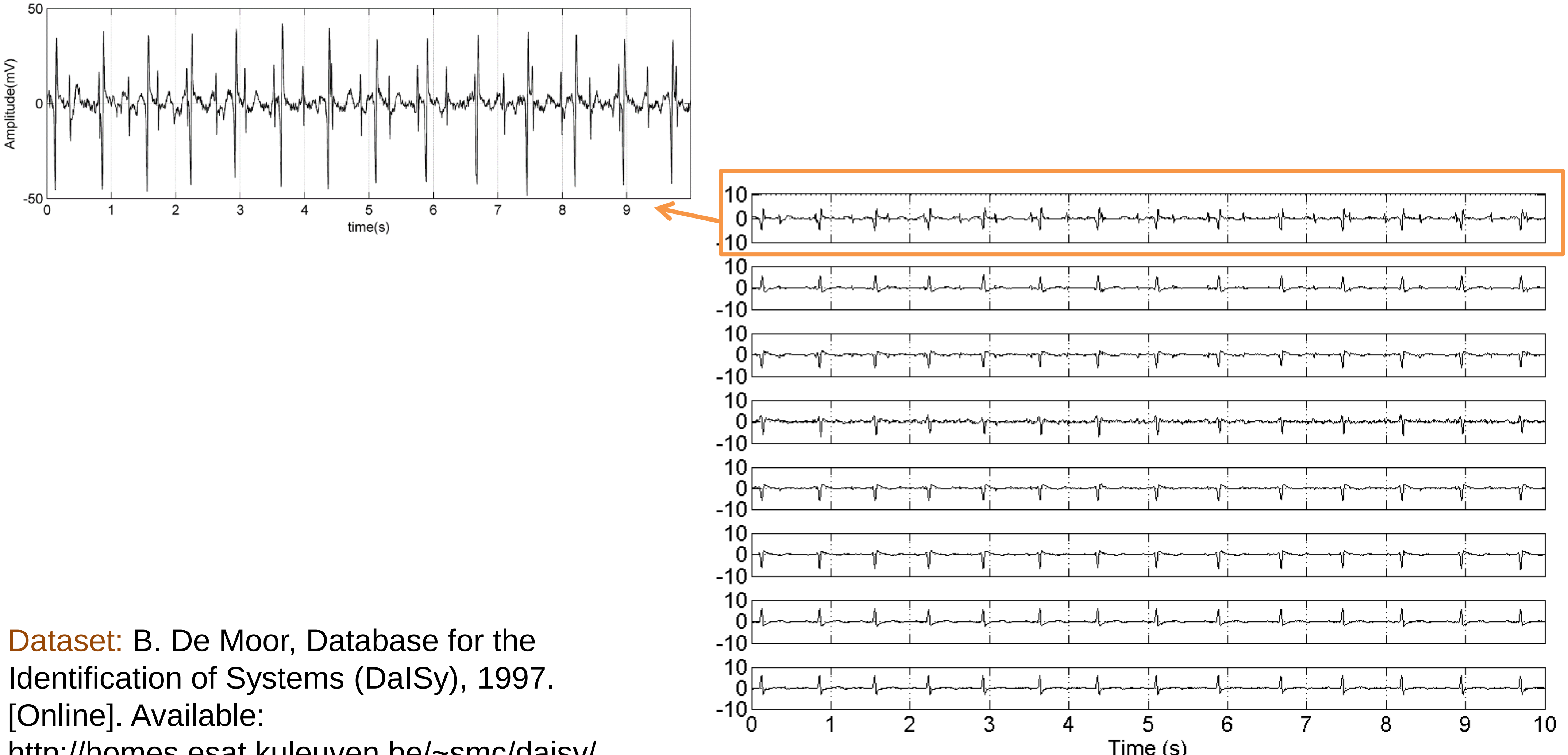
- The noises and interferences overlap with the fetal ECG in various domains





Typical Fetal ECG Samples

Example 1: The DaISy dataset



Dataset: B. De Moor, Database for the Identification of Systems (DaISy), 1997. [Online]. Available: http://homes.esat.kuleuven.be/~smc/daisy/

Typical Fetal ECG Samples

Example 2: Six-channel fetal ECG

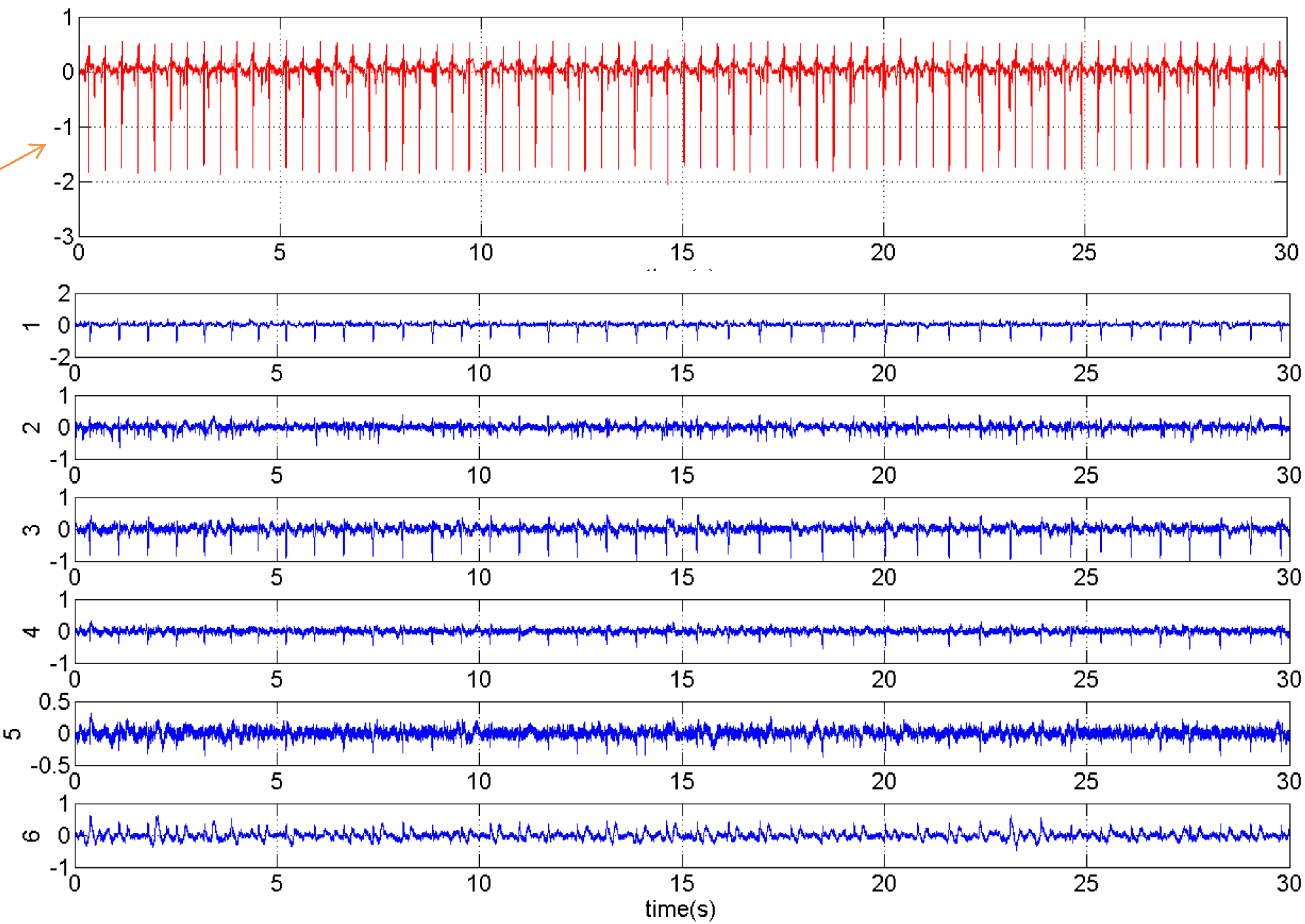
This dataset has been recorded by Dr. Evelyn Huhn and provided by Dr. Raphael Schneider

Typical Fetal ECG Samples

Example 3:

Fetal scalp lead

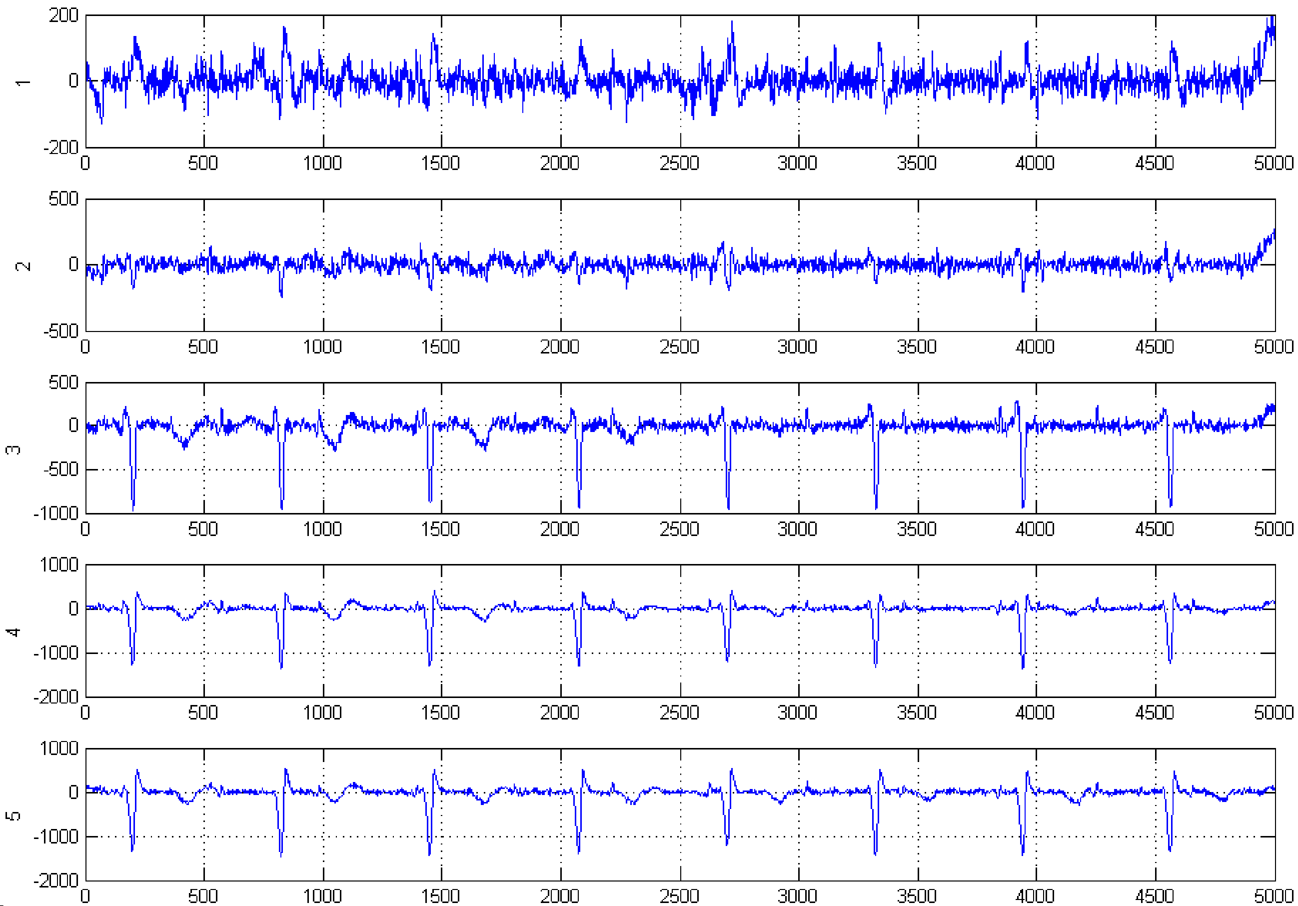
Maternal abdominal lead



Sample data provided by Dr. Adam Wolfberg, Tufts University

Typical Fetal MCG Samples

Example 4: Twin MCG recordings (5 out of 168 channels)



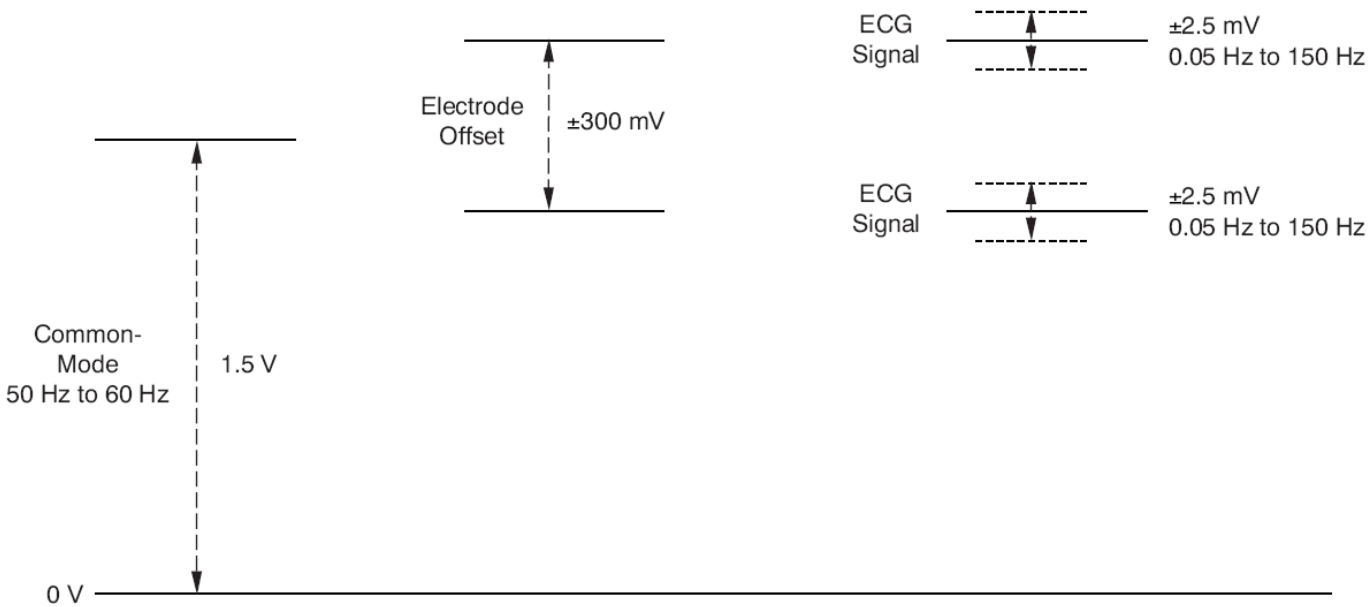
Sample data provided by Dr. Dirk Hoyer, Jena University

Digital Noninvasive Fetal ECG Acquisition

Analog and Digital Front-end Requirements

Fetal ECG Amplitude/Frequency Specifications

- The effective bandwidth of adult ECG is between 0.05Hz to 150Hz, with a maximum span of ±5mV in magnitude (besides the common-mode and electrode offset voltages)
- The front-end noise of adult ECG devices should be bellow 30µVRMS [IEC60601-2-51,27]
- Empirically the fetal ECG can be 10-20 times smaller than the mECG and wider in bandwidth: 0.05Hz to 250Hz (due to the sharper QRS and higher heart rate)
- fECG acquisition systems should have a broad dynamic range to enable fECG acquisition without overflow or saturation due to the mECG.



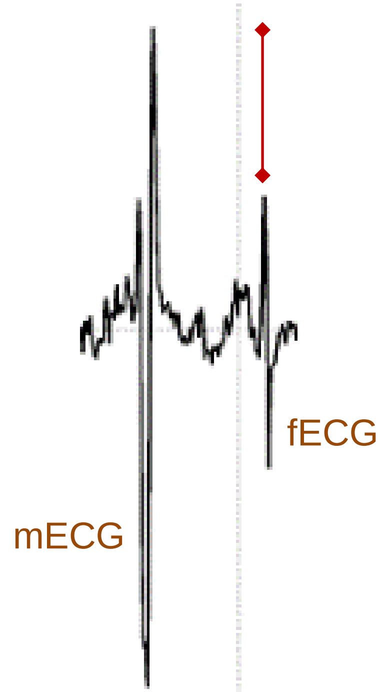


Fig. adopted from: Analog Front-End Design for ECG Systems Using Delta-Sigma ADCs, Texas Instruments, Application Report SBAA160A–Revision April 2010

Fetal ECG Digital Acquisition Requirements

No standards yet; but the recommended frontend specifications for fetal ECG acquisition is:

- Bandwidth (-3dB cutoff frequency):
  - Acceptable: 0.05Hz to 250Hz
  - Preferred: 0.05Hz to 1kHz (for better fECG-noise separability)
- Amplified analog voltage range: 3-5V (preferably in differential pairs)
- Analog-to-digital resolution:
  - Low resolution: 16bits
  - High resolution: 24bits
- Sampling frequency:
  - Minimum: 500Hz
  - Acceptable: 1kHz
  - High-resolution: 5-10kHz
- Sampling sequence:
  - Preferred: Simultaneous
  - Sequential (multiplexed): Acceptable only at high sampling frequencies

Remark: The sampling frequency should be above twice the maximum frequency of the input signal, known as the Nyquist rate to avoid aliasing and to guarantee information preservation. But for biomedical applications, sampling at the Nyquist rate itself does not give visually agreeable signals and the signals are commonly over-sampled above the Nyquist rate for better visualization (and possible SNR improvement).

Analog-to-Digital Converters Signal-to-Noise Ratio (SNR) – A REMINDER

- Standard SNR equation:

Over-Sampling Ratio (f_s/BW)

$$SNR_{dB} = 6.02B + 1.76dB + \underline{10\log_2(OSR)}$$

The nominal SNR with a full scale input sinusoidal input

The number of ADC bits

Obtained by post-filtering, if the signal is sampled above the minimal Nyquist rate

- Effective number of bits (ENOB):

$$ENOB = [TrueSNR_{dB} - 1.76dB]/6.02$$

The practical number of ADC bits (non-integer)

The true SNR obtained in practice (measurable)

- Note: ENOB is always smaller than B in practice (the ADC never achieves its nominal performance due to electronic noise)

Number of Channels

- Advanced fetal ECG extraction algorithms use multichannel maternal abdominal recordings
- There is no global standard in this domain yet; but devices with 1 to 72 channels have been developed and tested in the research community.
- Existing commercial devices use between 5 to 32 leads spread across the abdomen, the two sides and lower-back of pregnant women according to anatomical landmarks. Separate chest leads are sometimes used for recording the mECG directly (used as reference)
- Some researchers use conventional adult Ag/Cl disposable ECG electrodes, while commercial devices have designed custom electrodes with ultra low impedance

PART II

Noninvasive Fetal ECG Extraction Techniques

Fetal ECG Extraction & Analysis

State-of-the-Art

Fetal Electrocardiography History in a Glance

- 1906: M. Cremer observed the fetal ECG using string galvanometers [Cremer, 1906]
- 1950's: Improvements in measurement and amplification techniques [Lindsley, 1942]
- 1970's: Introduction of signal processing techniques in this domain [Farvet, 1968, Widrow et al., 1975]
- 1990's: Application of multichannel signal processing in this domain [van Oosterom, 1986, Kanjilal et al., 1997, Zarzoso et al., 1997]
- 2000's: Improvement of multichannel fECG acquisition and semi-blind source separation methods [Sameni, 2008]
- 2010-present: Algorithmic improvements and fECG parameter extraction (HR, QT, ST-segment, etc.) [Behar et al, 2014, 2016]

An Incomplete List of the Most Common fECG Extraction Techniques

- Direct Fetal ECG Analysis: Used in early studies; only possible in high signal-to-noise ratios by visual inspection [Larks, 1962]
- Adaptive and Matched Filtering: Partially effective; require reference electrodes [Farvet, 1968, Widrow et al., 1975, Park et al., 1992, Outram et al., 1995, Shao et al., 2004, Martens et al., 2007, Behar, 2016]
- Linear Decomposition: Rather effective; decompose the signals onto fixed or data-driven basis functions [Li et al., 1995, Khamene et al., 2000, Akay et al., 1996]
- Nonlinear Decomposition: Rather effective, but empirical; require prior information [Schreiber & Kaplan, 1996a, Schreiber & Kaplan, 1996b, Richter et al., 1998, Kantz & Schreiber, 1998, Kotas, 2004]
- Blind and Semi-Blind Source Separation: Very effective; require multiple channels [van Oosterom, 1986, Zarzoso et al., 1997, Cardoso, 1998, De Lathauwer et al., 2000] [Barros & Cichocki, 2001, Zhang & Yi, 2006, Li & Yi, 2008] [Vigneron et al., 2003, Jafari & Chambers, 2005] [Sameni, 2008, Biglari 2016, Fatemi 2017]

Preprocessing

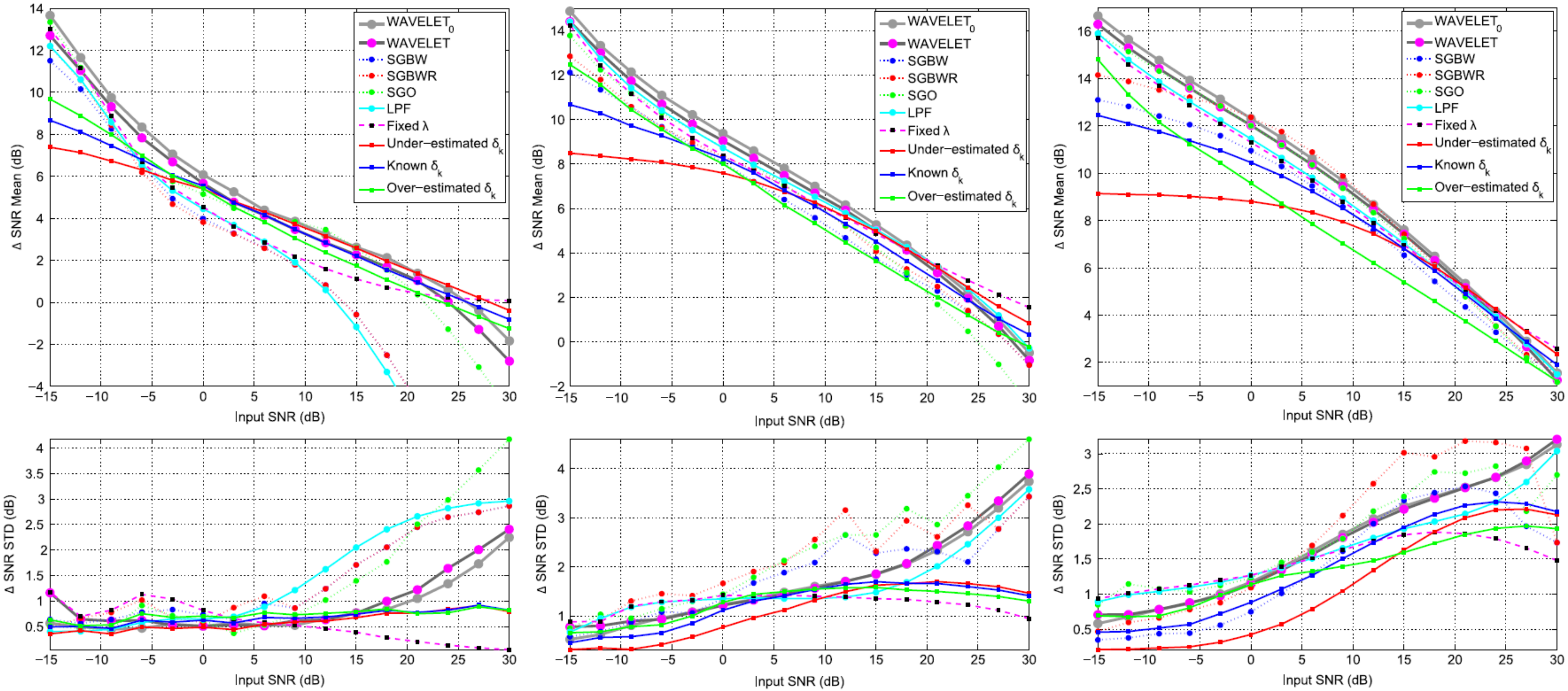
Power-Line, Device Noise, Out-of-Band Noise Rejection

Band-Pass Filtering

- Some fetal acquisition systems use DC-coupled ADC and higher upper cutoff frequencies (beyond 250Hz), which results in notable baseline wander and out-of-band noise.
- Power-line noise (50Hz or 60Hz) is also unavoidable, due to long lead cables (which are prone to inductive noises) and grounding issues
- These noises should be removed in the digital domain, before the main processing

Quantitative Comparison of Different Preprocessing Algorithms

The following SNR improvement results have been obtained over adults through an exhaustive Monte Carlo simulation in presence of additive white noise.



Normal Sinus Database; fs = 128Hz Arrhythmia database; fs = 360Hz PTB database; fs = 1kHz

Quantitative Comparison of Different Preprocessing Algorithms (continued)

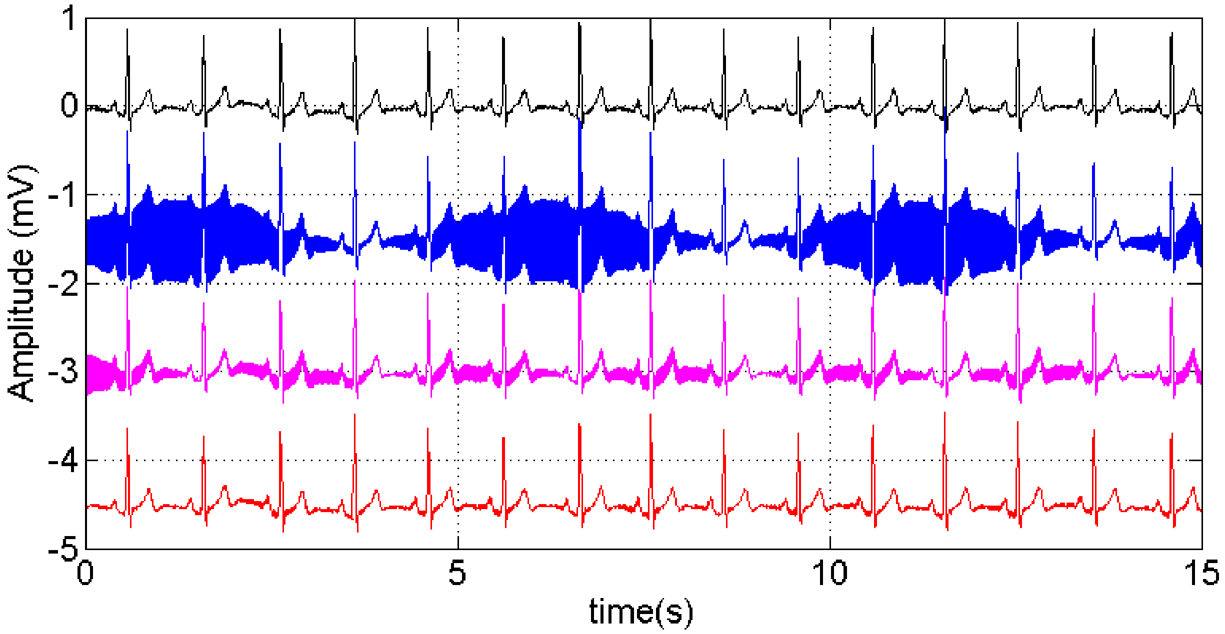
Some empirical rules-of-thumb for fetal/adult preprocessing filters:

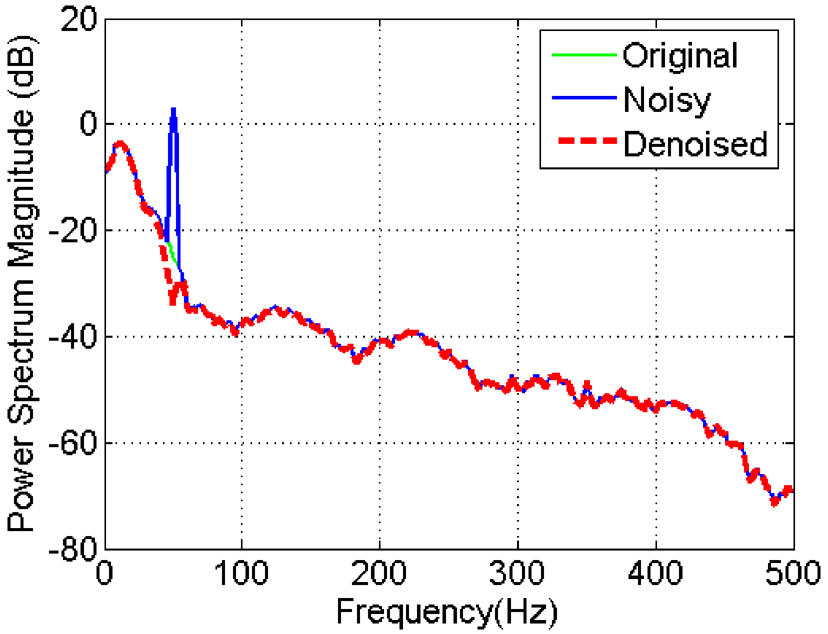
- If you require to obtain or pass the known ECG standards → stick to classical band-pass filtering (with greater consensus, which are more familiar for the medical/biomedical community)
- For biomedical research and **without any prior assumptions** or models (cyclostationarity, normality, anomaly, etc.) → use wavelets
- For biomedical research **with prior assumptions** on the background noise or ECG model → use model-based methods such as Kalman filters and matched filters

Ref: Sameni, R. (2017). Online filtering using piecewise smoothness priors: Application to normal and abnormal electrocardiogram denoising. *Signal Processing*, *133*, 52-63.

Adaptive Notch Filtering

- The power-line noise frequency is very stable (50Hz or 60Hz). However, inductive power-line noise magnitude can fluctuate due to maternal movement (any change in the cross sectional area of the inductive loop) or grounding issues with an unknown phase
- Adaptive filtering schemes using Kalman filters have been proposed for handling amplitude fluctuations of the power-line noise





Adopted from: R. Sameni, "A Linear Kalman Notch Filter for Power-Line Interference Cancellation," in *Proceedings of the 16th CSI International Symposium on Artificial Intelligence and Signal Processing (AISP)*, Shiraz, Iran, 2-3 May 2012 2012, pp. 604-610.

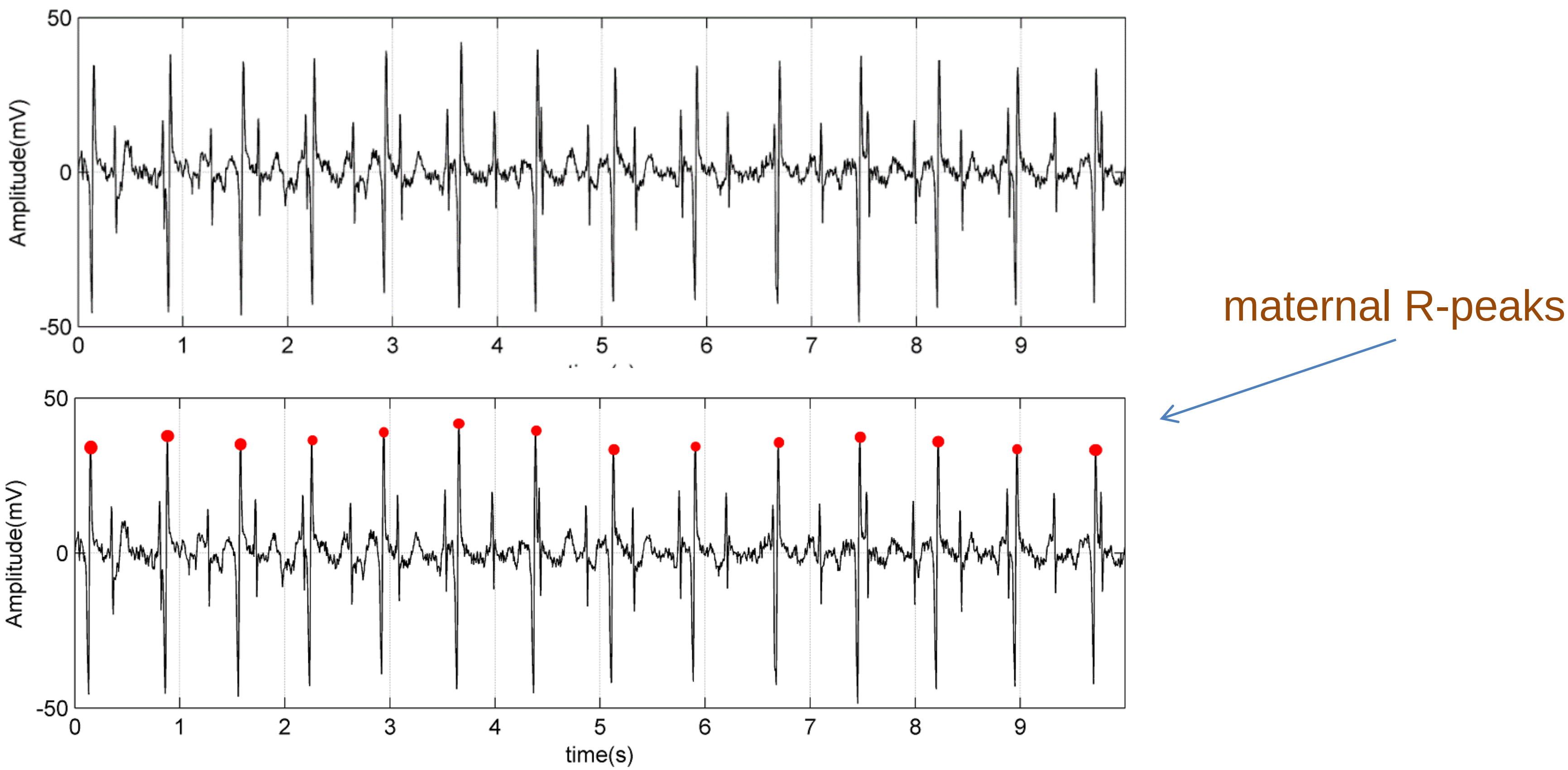
Basic Maternal ECG Cancellation and Fetal ECG Extraction Methods

Visual Inspection, Template Subtraction

fECG Detection by Naïve Visual Inspection

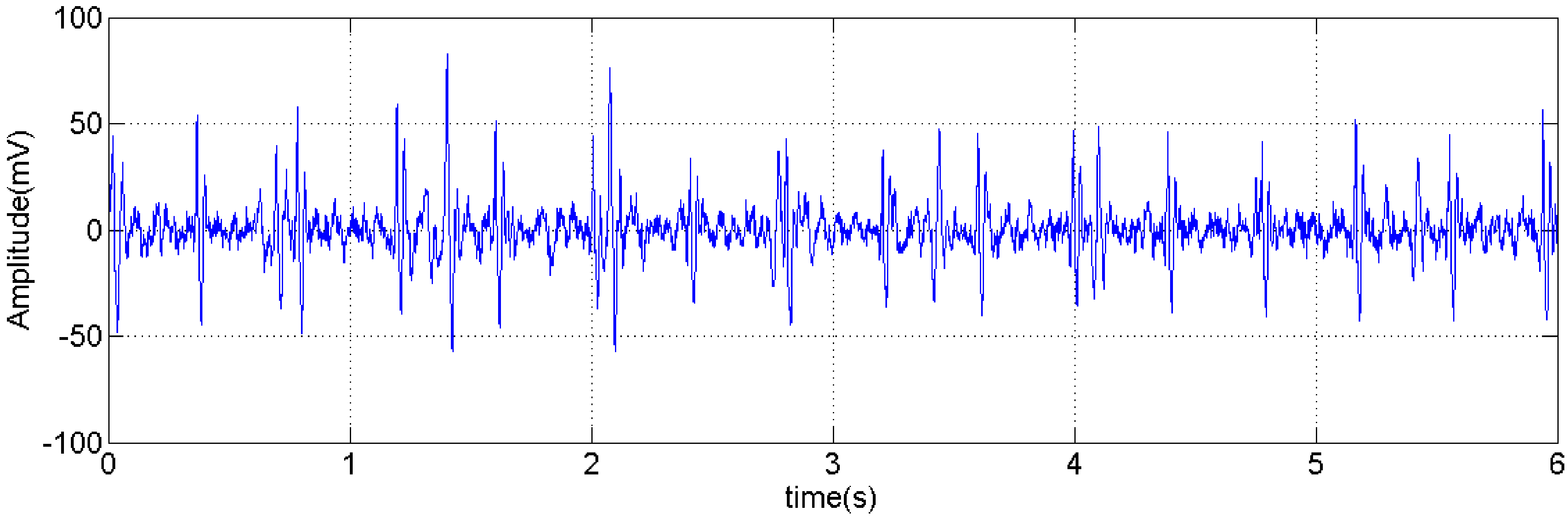
- The very first methods used for fECG extraction were (and are) based on basic visual inspection.
- The human eyes and brain are extremely powerful in pattern recognition.

Example 1:



fECG Detection by Naïve Visual Inspection
(continued)

Example 2: Can you spot the residual maternal peaks among the fetal peaks?



fECG Detection by Naïve Visual Inspection

(continued)

fECG detection by visual inspection is not always possible, as it is:

- Subjective: depends on the fetal position and electrode placement
- Unreliable and unsystematic: can not be automated and is not applicable to long data records
- Variable fECG morphology: Due to the nonstandard acquisition method, does not lead into a canonical (standard) representation of the fetal ECG waveform

✓ Nevertheless, it remains as the most basic step for fECG signal quality assessment

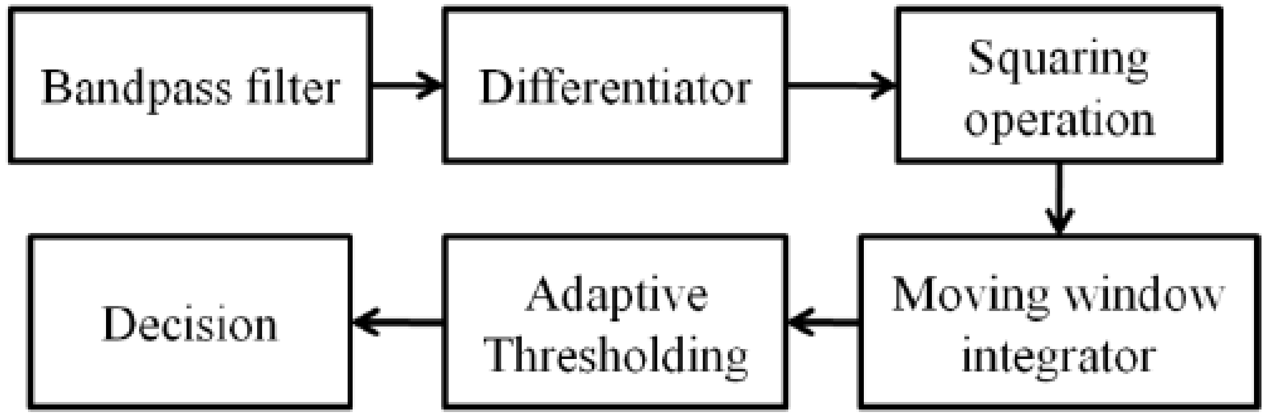
Maternal R-Peak Detection

- The first step in most fetal ECG extraction algorithms is to detect the maternal R-peaks (as the major interference for the fECG)
- As noted in the introduction, all ECG channels are time-aligned (despite their morphologic variations)
- Hence, the maternal R-peaks can be estimated from the abdominal leads or from an accessory maternal chest lead.
- Since the mECG is strong, any R-peak detection method is acceptable at this stage: local peak search, Pan-Tompkins, etc. (we'll present more robust methods, when it comes to the fECG heart-rate calculation)

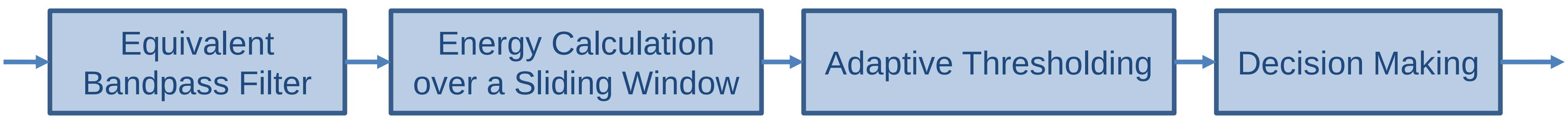
The Pan-Tompkins R-Peak Detector

- A classical method for R-peak detection has been proposed by J Pan and WJ Tompkins in 1985.

Basic block diagram:



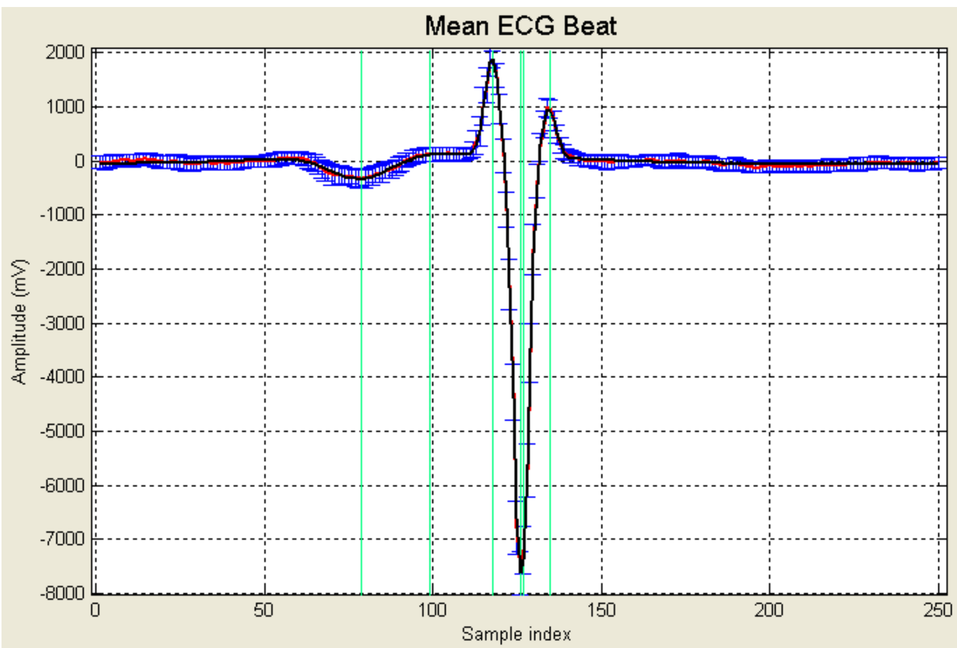
Simplified diagram:



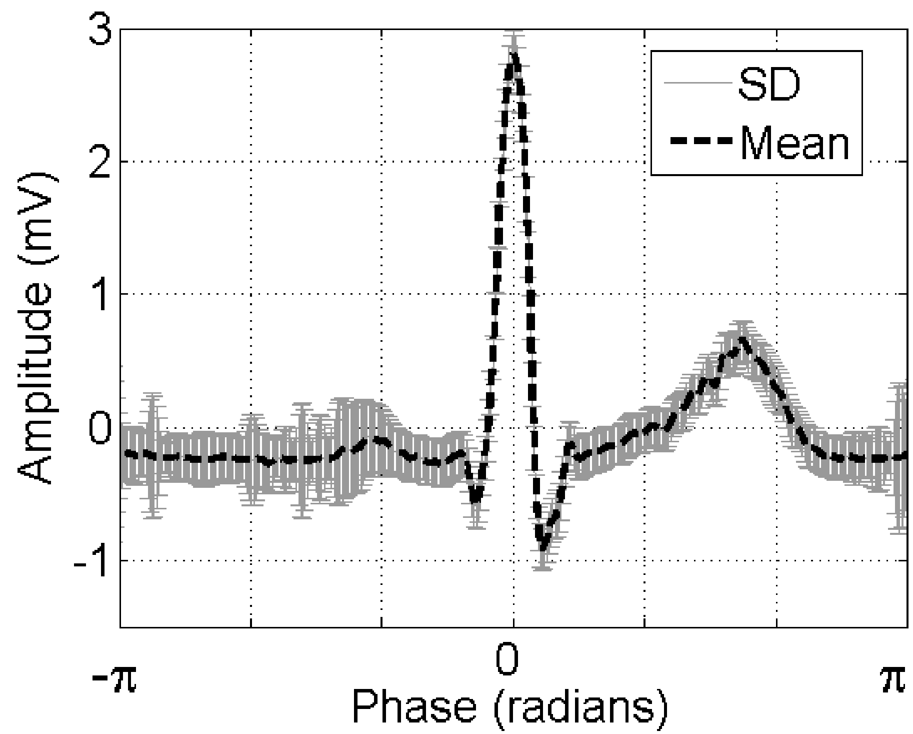
The Pan-Tompkins method: Pan, Jiapu, and Willis J. Tompkins. "A real-time QRS detection algorithm." *IEEE Trans. Biomed. Eng* 32.3 (1985): 230-236.

Average Maternal ECG Morphology

- By aligning the maternal R-peaks under each other, the so-called waterfall plot and the average mECG morphology is obtained



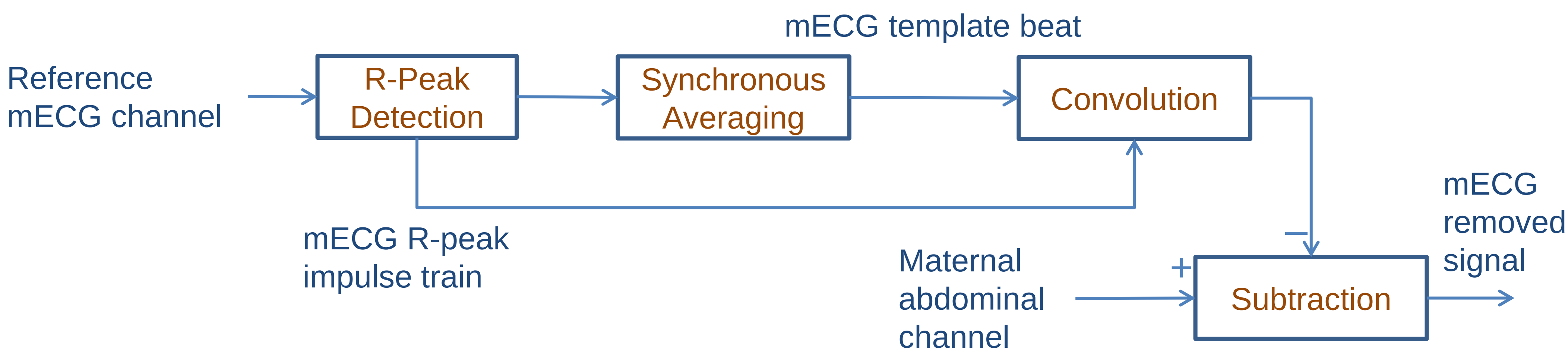
Sample-wise averaging



Phase-based averaging

Maternal ECG Cancellation by Template Subtraction

- A basic approach for mECG cancellation is to form a template of the mECG (from a reference chest channel or the abdominal channels) and to subtract this template from the abdominal leads (synchronous with the R-peaks)



Maternal ECG Cancellation by Template Subtraction (continued)

Template subtraction issues:

1. The calculation of the mECG template is prone to noise and beat-wise variations
2. The reference mECG channel is not morphologically identical to the mECG superposed over the abdominal leads
3. The mECG morphology varies from beat to beat and with heart-rate variations
4. Issues with abnormal mECG beats (do not resemble the normal beats)
5. The mECG and fECG may exactly overlap in time

We propose solutions for each issue in what follows

Improved Maternal Template Calculation

- The maternal ECG template can be made more robust by using weighted averaging
- Assuming N ECG beats as $x_i[k] = s[k] + n_i[k]$ $(i = 1, \ldots, N)$, where $s[k]$ is the desired template beat and $n_i[k]$ is the beat-wise noise, the weighted average is:

$$v[k] = \sum_{i=1}^{N} w_i x_i[k]$$

where $w_i = [\sum_{j=1}^{N} \frac{\sigma_i^2}{\sigma_j^2}]^{-1}$ and σ_i^2 denotes the noise variance of the segment i.

- Intuition: “give less weight to the more noisy beats”

Ref: J. Leski,“Robust weighted averaging [of biomedical signals],”IEEE Trans. Biomed. Eng., vol. 49, no. 8, pp. 796–804, 2002.

Vectorcardiogram Loop Alignment

- In the context of adult ECG analysis, VCG loop alignment has been proposed for more accurate alignment and estimation of average ECG beats. The same tools can be used for fetal ECG.
- In this method, each beat Z is assumed to be a scaled, rotated and translated version of a reference beat Z_R, plus noise:

$$Z = \alpha Q Z_R J_\tau + e$$

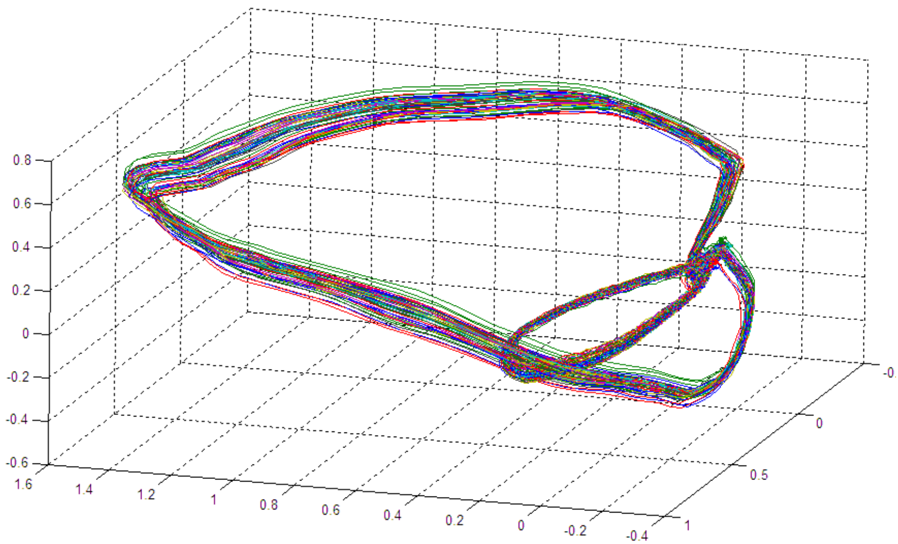
- The scaling, rotation and translation is next found by solving a least-squares error estimation problem: $[\alpha, Q, J_\tau] = \text{argmin} ||Z - \alpha Q Z_R J_\tau||^2$

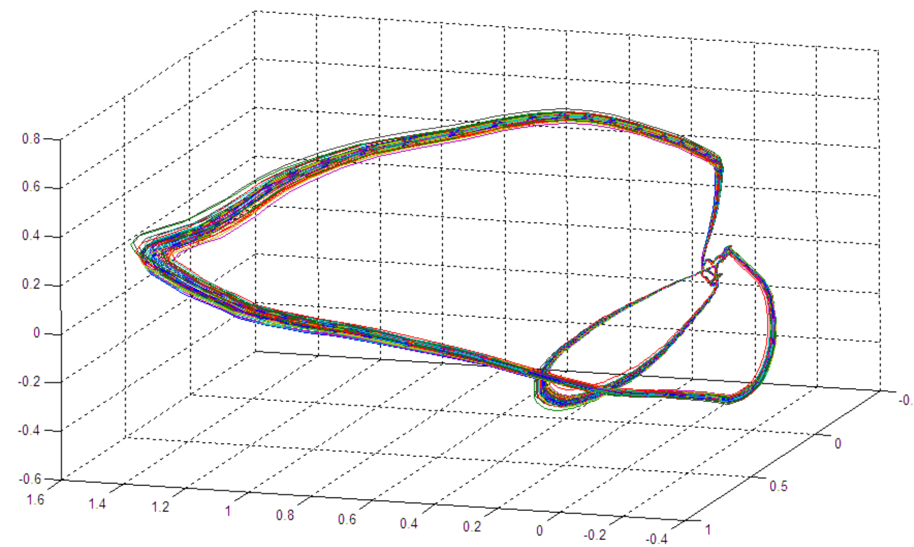
Refs:

- L. Sornmo, “Vectorcardiographic loop alignment and morphologic beat-to-beat variability,” Biomedical Engineering, IEEE Transactions on, vol. 45, no. 12, pp. 1401–1413, December 1998.
- M. Astrom, E. Santos, L. Sornmo, P. Laguna, and B. Wohlfart, “Vectorcardiographic loop alignment and the measurement of morphologic beat-to-beat variability in noisy signals,” Biomedical Engineering, IEEE Transactions on, vol. 47, no. 4, pp. 497–506, April 2000
- Vullings, R., Peters, C. H. L., Mischi, M., Oei, S. G., & Bergmans, J. W. M. (2008, August). Fetal movement quantification by fetal vectorcardiography: a preliminary study. In *Engineering in Medicine and Biology Society, 2008. EMBS 2008. 30th Annual International Conference of the IEEE* (pp. 1056-1059). IEEE

Vectorcardiogram Loop Alignment

Example: VCG loop alignment in adults





VCG loop before alignment

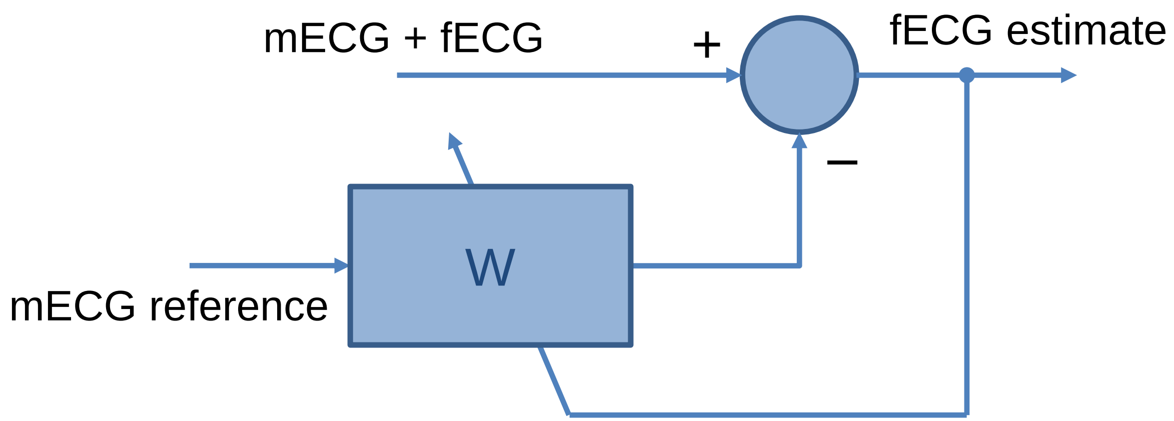
VCG loop after alignment
(notice the smaller loop variance)

Adaptive Maternal ECG Cancellation Methods

Adaptive Filters

Adaptive Filters for mECG Cancellation

Different types of adaptive filters have been used for mECG cancellation from maternal abdominal recordings. The most basic scheme is as follows:



Ref: N. V. Thakor and Y. S. Zhu, “Application of adaptive filtering to ECG analysis: noise cancellation and arrhythmia detection,” IEEE Trans. Biomed. Eng., vol. 38, pp. 785–794, 1991.

Adaptive Filters for mECG Cancellation
(continued)

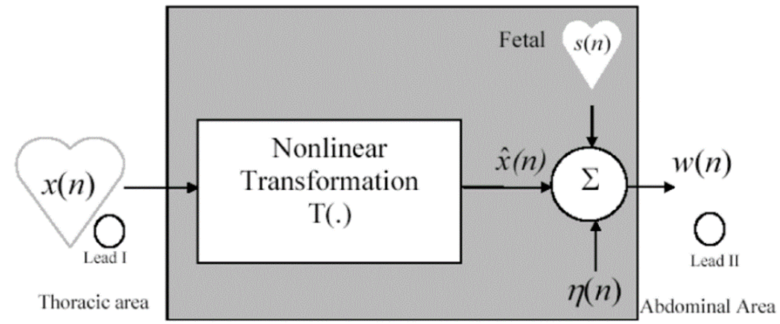
Limitations of adaptive filters for mECG cancellation:

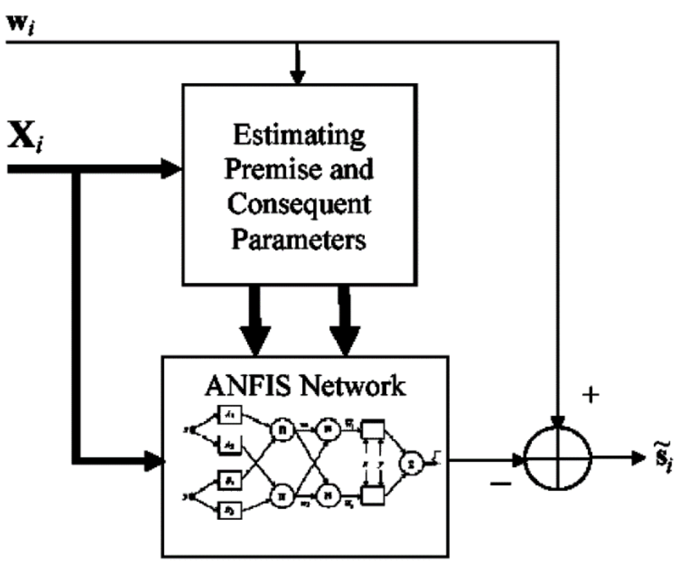
1. It requires a reference mECG channel, which is a noiseless copy of the interfering mECG; while the maternal chest leads are morphologically different from the abdominal interference.
2. The weight update is based on least square error (LSE) estimation. This measure does not encounter the existence of the fECG or any prior knowledge of the background noise probability distribution.

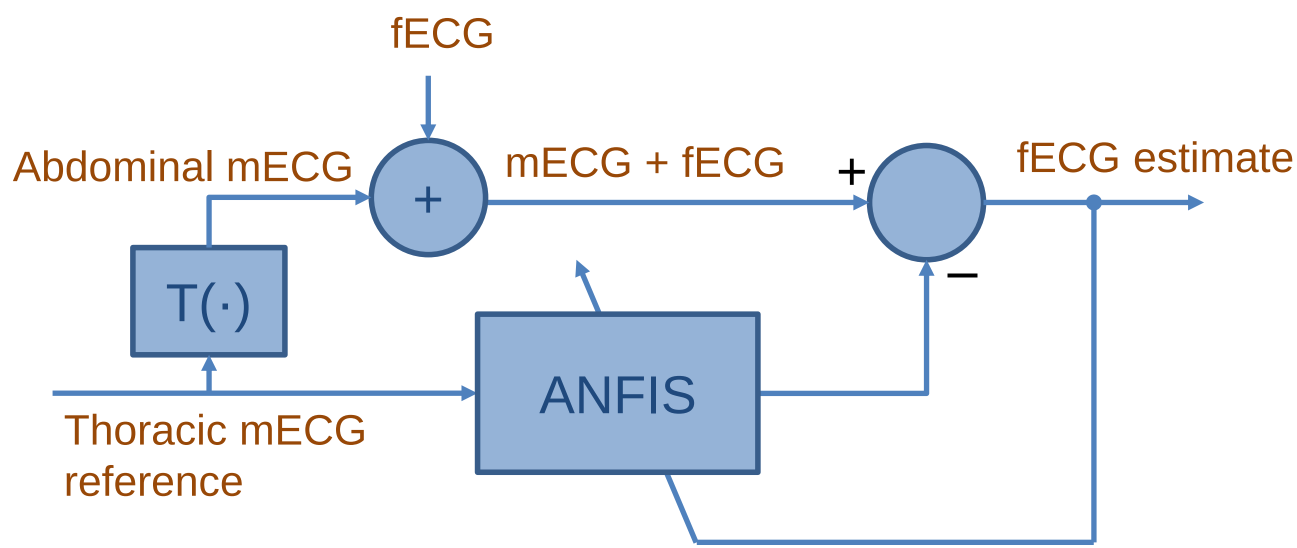
To overcome these limitations, we can switch to model-based filtering approaches, which require morphological models of the mECG beats

Maternal Abdominal mECG Reconstruction

One of the methods for solving the issue of different ECG morphologies is by using nonlinear transformed versions of thoracic lead(s). This needs more than one reference channel for good performance (due to the morphologic diversity)







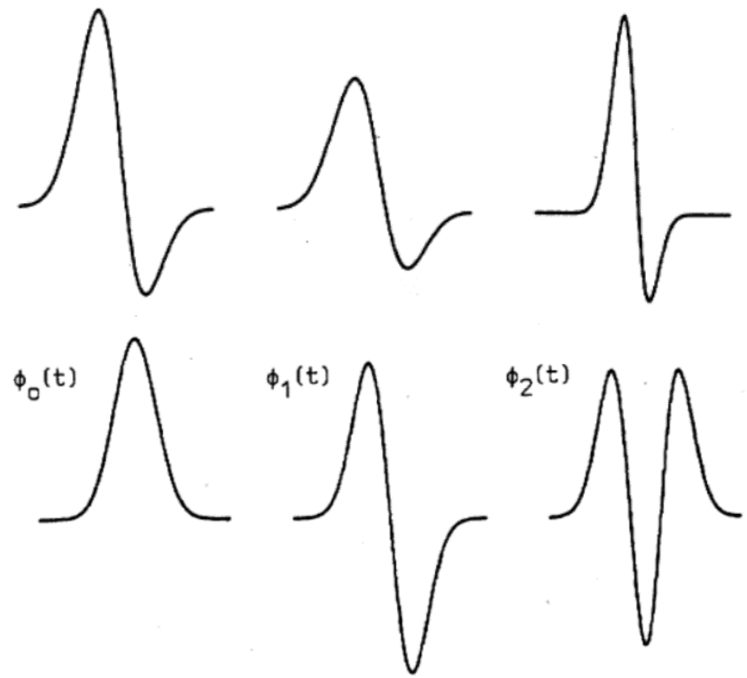
Refer to: Assaleh, K. (2007). Extraction of fetal electrocardiogram using adaptive neuro-fuzzy inference systems. *IEEE Transactions on Biomedical Engineering*, *54*(1), 59-68.

Model-Based Maternal ECG Cancellation Methods

Kalman Filters

Morphologic ECG Models

- Mathematical modeling of the ECG has many applications in ECG test instruments and pedagogy
- It dates back to the 1960's:
  - Scher et al. (1960) → Factor Analysis
  - Sornmo et al. (1981)→ Bessel Functions
  - Laguna et al. (1996) → Hermite polynomials
  - McSharry et al. (2003)→ Gaussian kernels in a dynamical form



Fetal ECG modeling: T. Oostendorp, *Modeling the Fetal ECG*. Ph.D. dissertation, K. U. Nijmegen, The Netherlands, 1989

McSharry-Clifford's model became a turning point in ECG modeling and analysis; due to its dynamic representation

McSharry-Clifford's Synthetic ECG Generator and its Polar Extension

A dynamic model of the ECG using sum of Gaussian functions:

$$\begin{cases} \dot{x} = \rho x - \omega y \\ \dot{y} = \rho y + \omega x \\ \dot{z} = -\sum\limits_{i \in \{P,Q,R,S,T\}} a_i \Delta\theta_i exp(-\frac{\Delta\theta_i^2}{2b_i^2}) - (z - z_0) \end{cases}$$

$$\rho = 1 - \sqrt{x^2 + y^2}$$

$$\theta = atan2(y, x)$$

$$\Delta\theta_i = (\theta - \theta_i)mod(2\pi)$$

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta)mod(2\pi) \\ z_{k+1} = -\sum\limits_i \delta \frac{\alpha_i \omega}{b_i^2} \Delta\theta_i exp(-\frac{\Delta\theta_i^2}{2b_i^2}) + z_k + \eta \end{cases}$$

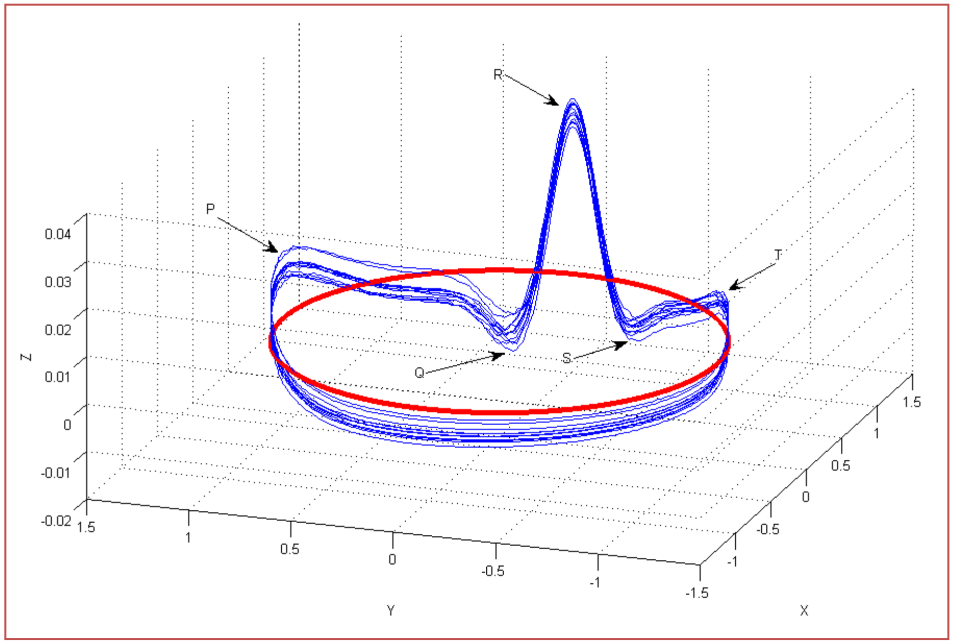
$$\Delta\theta_i = (\theta_k - \theta_i)mod(2\pi)$$

Ref:

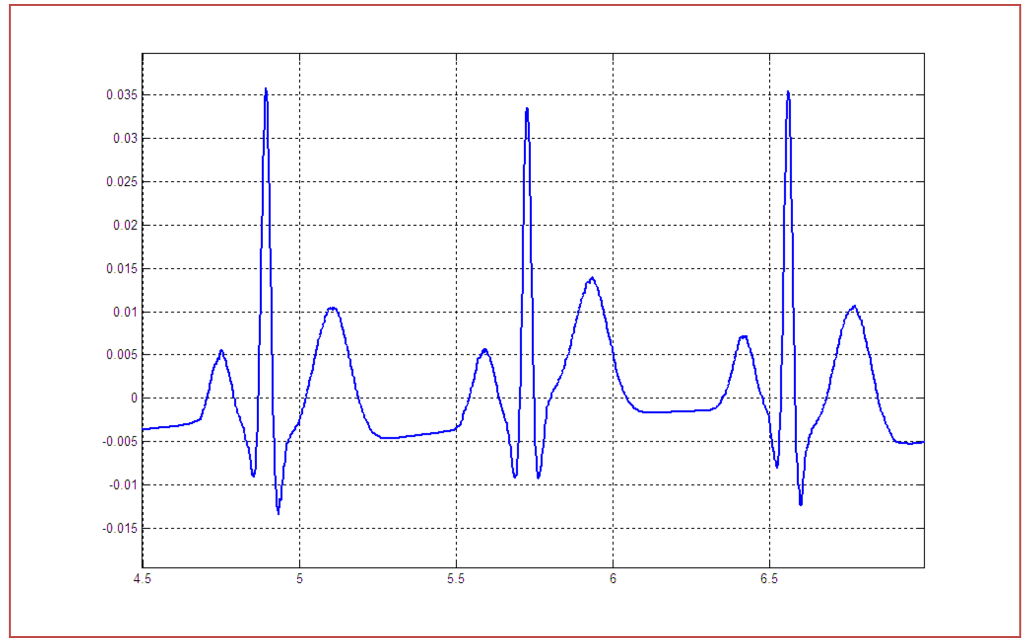
- P.E. McSharry, et al. "A dynamical model for generating synthetic electrocardiogram signals" IEEE Transactions on Biomedical Engineering 50.3 (2003): 289-294.
- R. Sameni, et al. "Filtering noisy ECG signals using the extended Kalman filter based on a modified dynamic ECG model." Computers in Cardiology, 2005. IEEE, 2005.

McSharry-Clifford’s Synthetic ECG Generator

Example:



3D trajectory of McSharry-Clifford’s model



A sample synthetic ECG

Further Extensions of McSharry-Clifford's Model

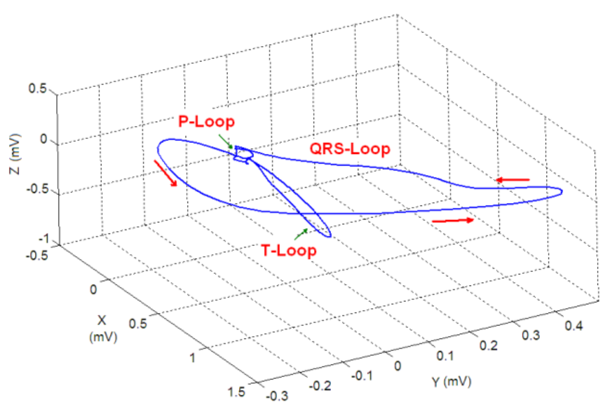
- McSharry-Clifford's ECG model was further extended to
  - Multichannel adult and fetal VCG and ECG
  - Normal versus abnormal ECG with ectopic beats

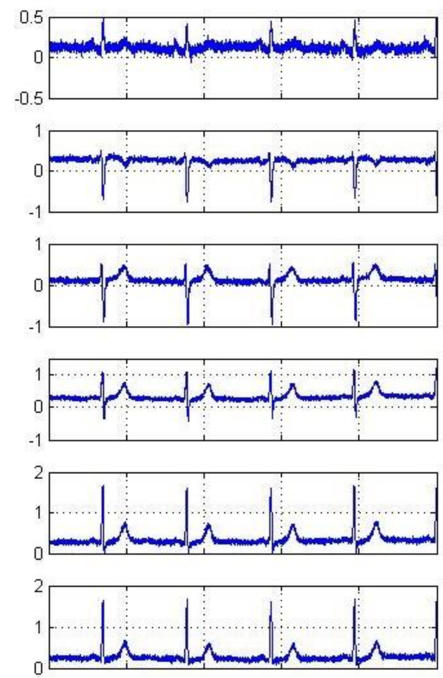
$$\dot{\theta} = \omega$$

$$\dot{x} = -\sum_i \frac{\alpha_i^x \omega}{(b_i^x)^2} \Delta\theta_i^x exp[-\frac{(\Delta\theta_i^x)^2}{2(b_i^x)^2}]$$

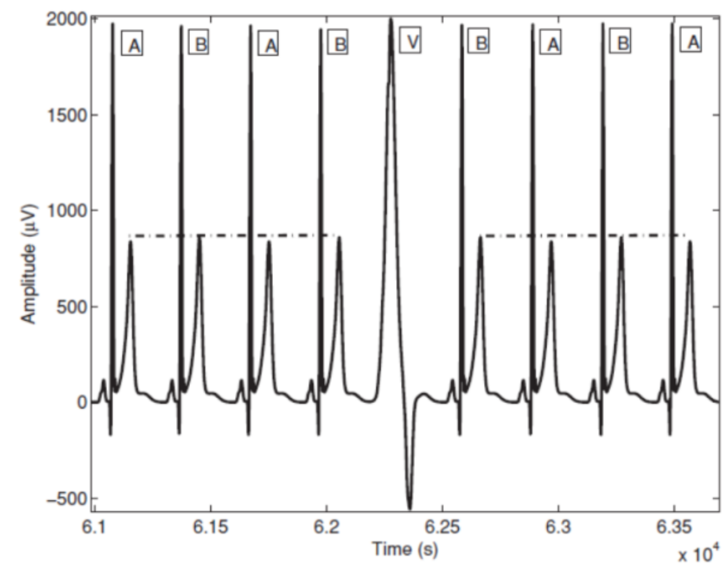
$$\dot{y} = -\sum_i \frac{\alpha_i^y \omega}{(b_i^y)^2} \Delta\theta_i^y exp[-\frac{(\Delta\theta_i^y)^2}{2(b_i^y)^2}]$$

$$\dot{z} = -\sum_i \frac{\alpha_i^z \omega}{(b_i^z)^2} \Delta\theta_i^z exp[-\frac{(\Delta\theta_i^z)^2}{2(b_i^z)^2}]$$





(Sameni et al. 2007)



(Clifford et al. 2010)

A Kalman Filter for ECG Denoising

Dynamical models of the ECG can be used to design a Kalman filter (or extended Kalman filter) for ECG denoising:

Process equation:

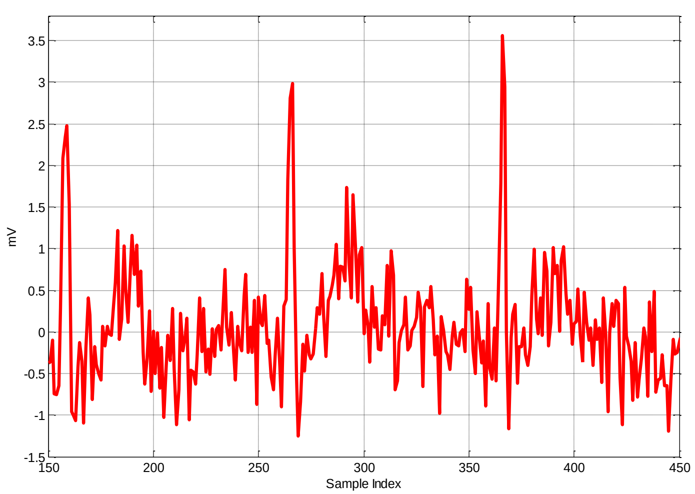
$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta) mod(2\pi) \\ s_{k+1} = -\sum_{i=1}^{N} \delta \frac{\alpha_i \omega}{b_i^2} \Delta\theta_i exp(-\frac{\Delta\theta_i^2}{2b_i^2}) + s_k + \eta \end{cases}$$

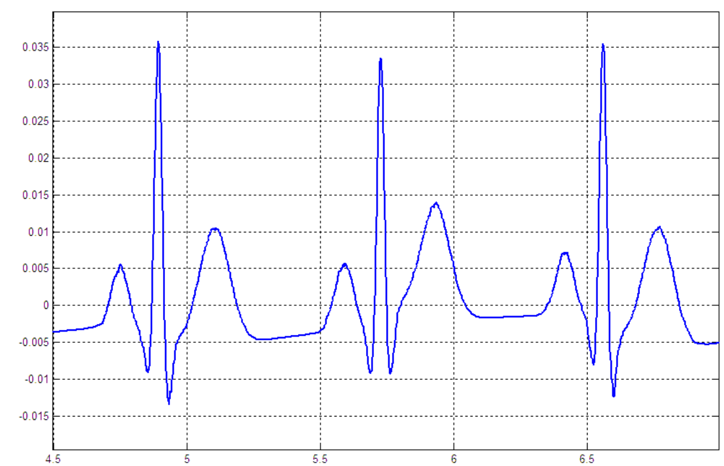
Dynamical ECG model

Observation equation:

$$\begin{cases} \phi_k = \theta_k + u_k \\ y_k = s_k + v_k, \end{cases}$$

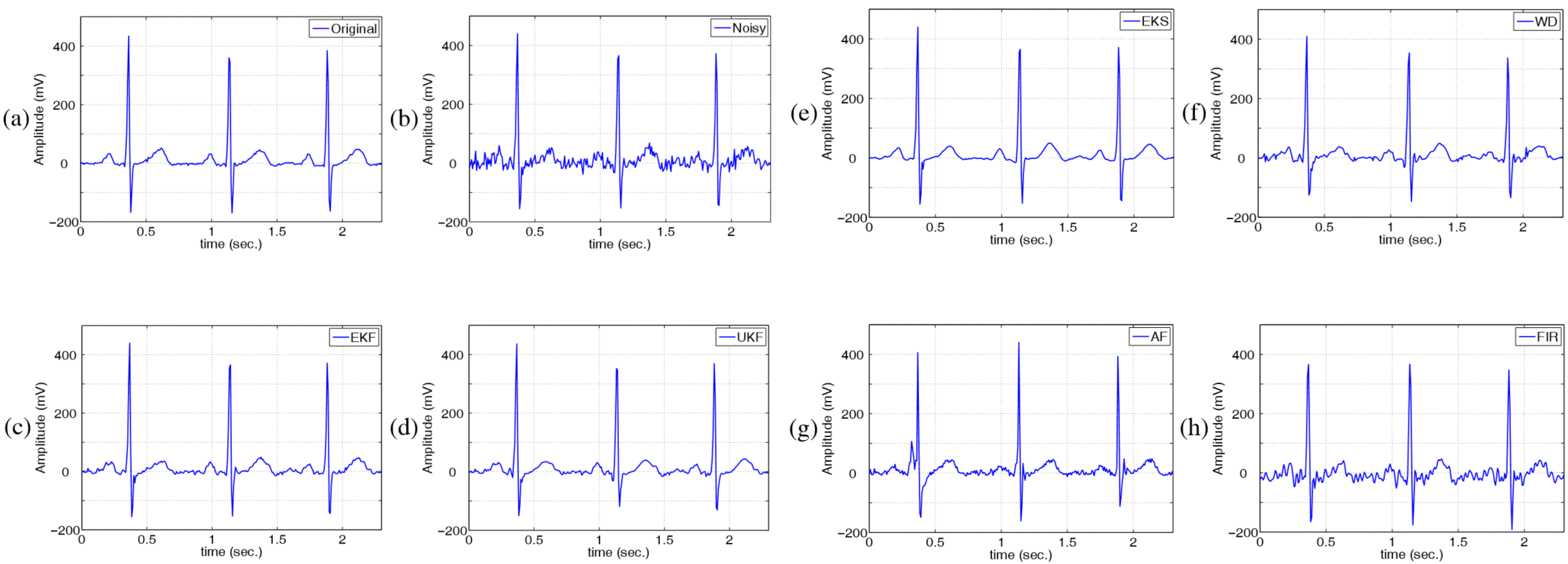
Noisy observations





A Kalman Filter for ECG Denoising

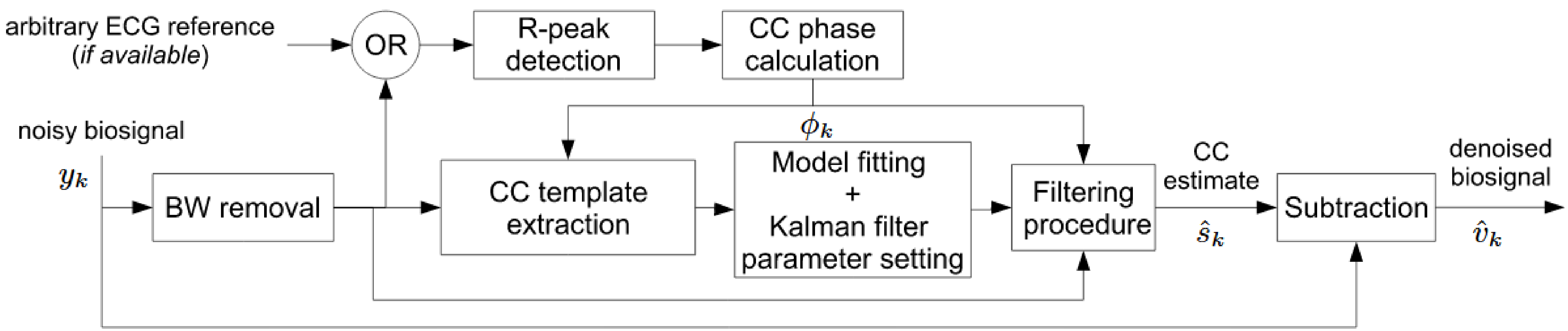
Example:



Ref: R. Sameni, et al. "A nonlinear Bayesian filtering framework for ECG denoising." IEEE Transactions on Biomedical Engineering 54.12 (2007): 2172-2185.

Maternal ECG Cancellation using Kalman Filters

For our application of interest, the noted Kalman filtering scheme can be used to adaptively remove the mECG from maternal abdominal recordings:



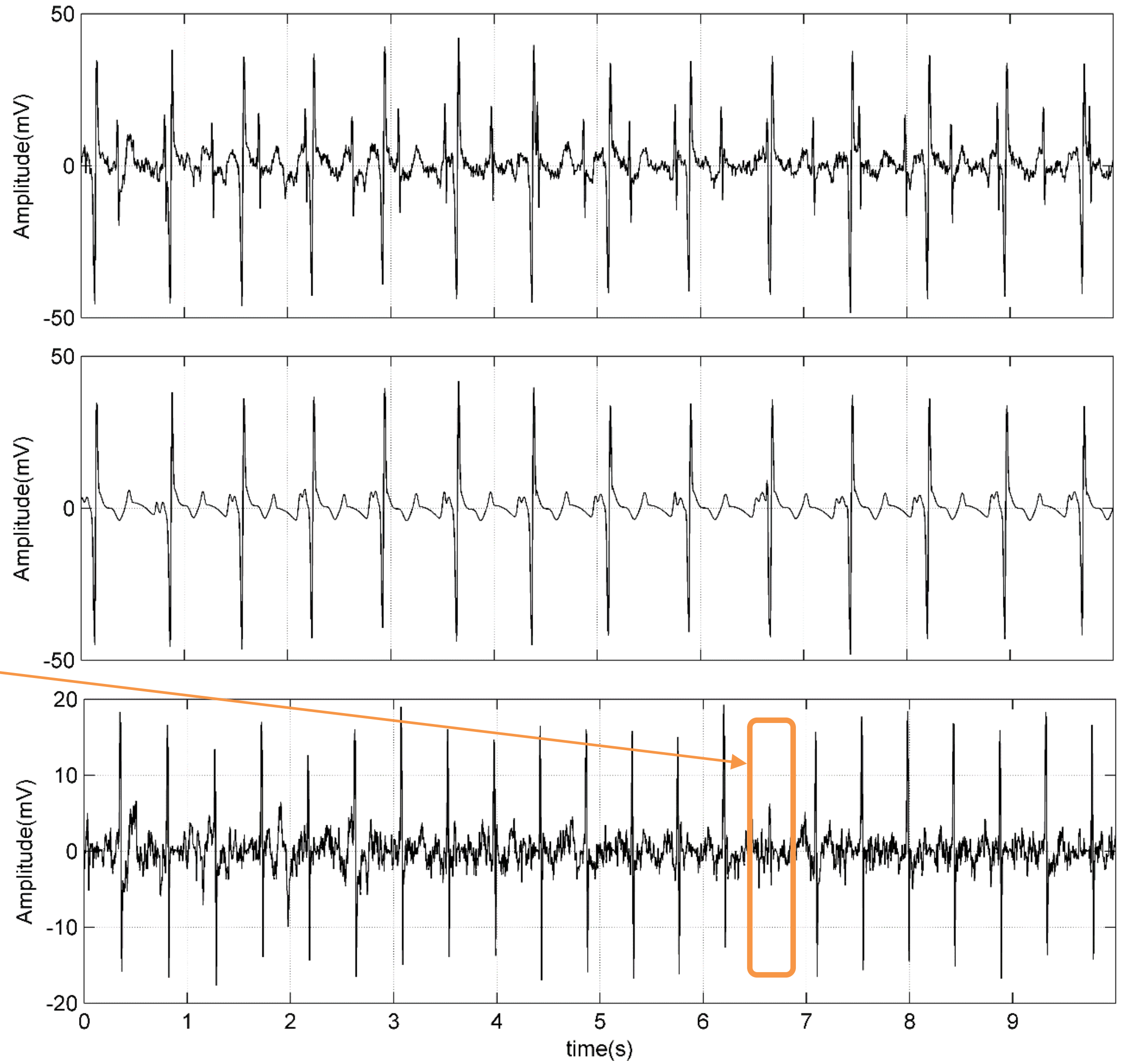
Refs:

• R. Sameni, M.B. Shamsollahi, and C. Jutten (2008). Model-based Bayesian filtering of cardiac contaminants from biomedical recordings. *Physiological Measurement*, *29*(5), 595.

•R. Sameni. "Extraction of fetal cardiac signals from an array of maternal abdominal recordings." PhD diss., Institut National Polytechnique de Grenoble-INPG; Sharif University of Technology (SUT), 2008.

Maternal ECG Cancellation using Kalman Filters

Example:



Original maternal abdominal signal

-

Estimated maternal ECG using Kalman filter

=

Residual fetal ECG plus noise

fECG removed due to temporal maternal and fetal QRS overlap

Maternal-Fetal QRS Overlap

- There is no temporal synchrony between the maternal and fetal ECG. Therefore, the QRS complex of the mother frequently overlaps with the fetal QRS over time
- Single-channel Kalman filters may lose the fECG during such epochs
- Multichannel processing is a solution, with the following advantages:
  - Uses the spatial diversity of the channels to separate the fetal and maternal ECGs
  - Provides a multiple perspective of the fetal heart

✓ Blind and semi-blind source separation methods are fundamental techniques in this domain

Blind and Semi-Blind Source Separation

An Overview

Mathematical Problem Definition

Data Model: In the most general case, blind source separation (BSS) can be mathematically formulated as follows:

$$\mathbf{x}(t) = \mathbf{f}(\mathbf{s}(t),t)$$

where

- t denotes time
- $\mathbf{x}(t)$ are N channels of observations (known)
- $\mathbf{s}(t)$ are M channels of sources (unknown)
- $\mathbf{f}(\cdot,t)$ is a (generally) nonlinear and time-variant transform (unknown)

Objective: Estimate $\mathbf{s}(t)$ from $\mathbf{x}(t)$, using minimal assumptions on the sources and mixtures (this is perhaps what the prefix “blind” stands for)

Blind Source Separation Solutions

- The problem is apparently ill-posed and without any other prior assumptions, there are no unique solutions
- Various cases of BSS with proven solutions include:

Linear mixtures:

- Statistically independent and non-Gaussian sources
- Signals with temporal correlation
- Sparse sources (in various domains)
- Temporally nonstationary sources
- Periodic sources

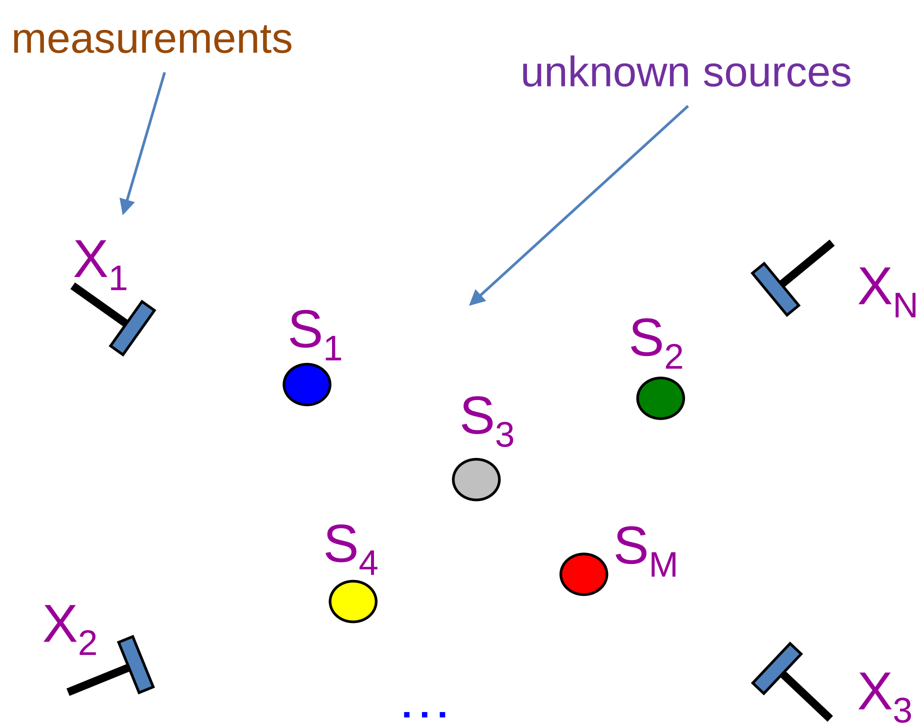
Nonlinear mixtures:

- Post-nonlinear sources
- Signals with slow temporal variations
- Sparse sources

Linear Mixtures of Independent Sources

Data Model: The most classical case of BSS with a proven solution is for linear mixtures of independent sources (random variables of random processes).

Linear Model: $\mathbf{x_i} = \mathbf{a_{i,1}s_1} + \mathbf{a_{i,2}s_2} + \mathbf{\ldots} + \mathbf{a_{i,M}s_M}, \quad \mathbf{i} = \mathbf{1,2,\ldots,N}$



$$\begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \\ \mathbf{x_N} \end{bmatrix} = \begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & & \mathbf{a_{1M}} \\ \mathbf{a_{21}} & . & & . \\ & & . & . \\ \mathbf{a_{N1}} & . & . & \mathbf{a_{NM}} \end{bmatrix} . \begin{bmatrix} \mathbf{s_1} \\ \mathbf{s_2} \\ \\ \mathbf{s_M} \end{bmatrix}$$

or in matrix form: $\mathbf{X} = \mathbf{AS}$

In the noisy case: $\mathbf{X} = \mathbf{A} \cdot \mathbf{S} + \mathbf{n}$ noise

Linear Mixtures of Independent Sources (continued)

- N = M: Determined
- N > M: Over-determined
- N < M: Under-determined (degenerate)

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & & \mathbf{a}_{1M} \\ \mathbf{a}_{21} & . & & . \\ & & . & . \\ \mathbf{a}_{N1} & . & . & \mathbf{a}_{NM} \end{bmatrix} . \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \\ \mathbf{s}_M \end{bmatrix}$$

- The measurements and the sources can generally be random processes rather random variables, i.e., **x**(t) and **s**(t).
- Independent Component Analysis (ICA) is one of the possible solutions of BSS, which assumes mixtures of statistically independent sources
- In order to see how classical linear ICA works, we need to see how linear transforms alter the topology of random variables

Linear Blind Source Separation Solutions

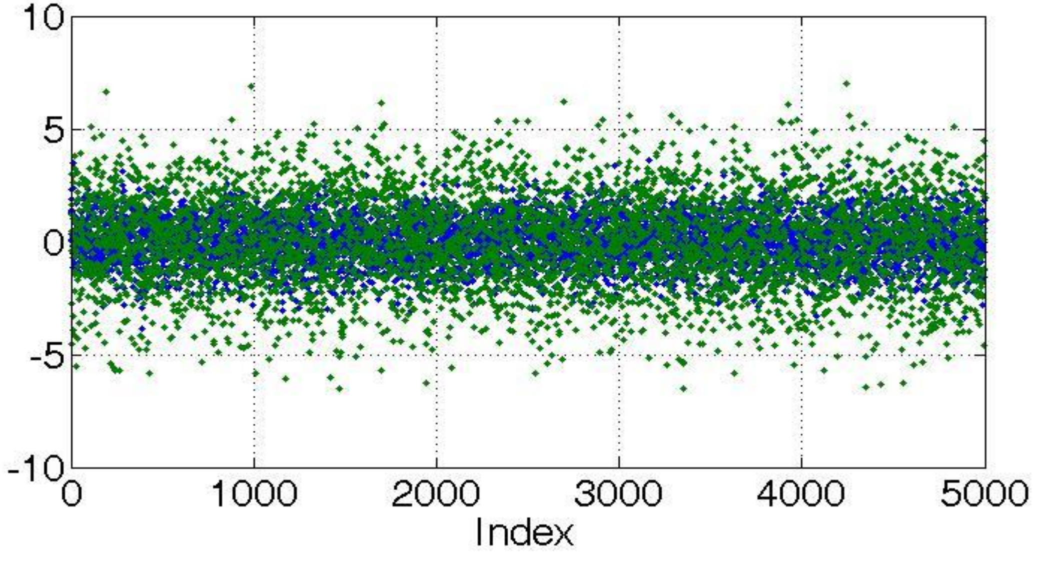
- In what follows, we study two approaches used for linear BSS without noise:
  1. Iterative methods (such as fastICA)
  2. Algebraic methods (such as AMUSE and ΠCA)

These methods represent two major classes of BSS methods, which have been used for noninvasive fetal ECG extraction

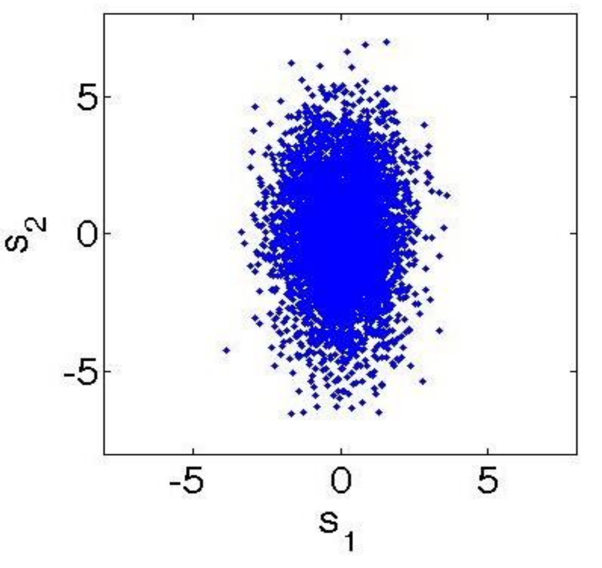
Linear Transforms of Gaussian Random Variables

We will present the idea behind ICA with a basic example:

Example 1: Suppose that s_1 and s_2 be independent Gaussian random variables with $s_1 \sim N(0,1)$ and $s_2 \sim N(0,2)$



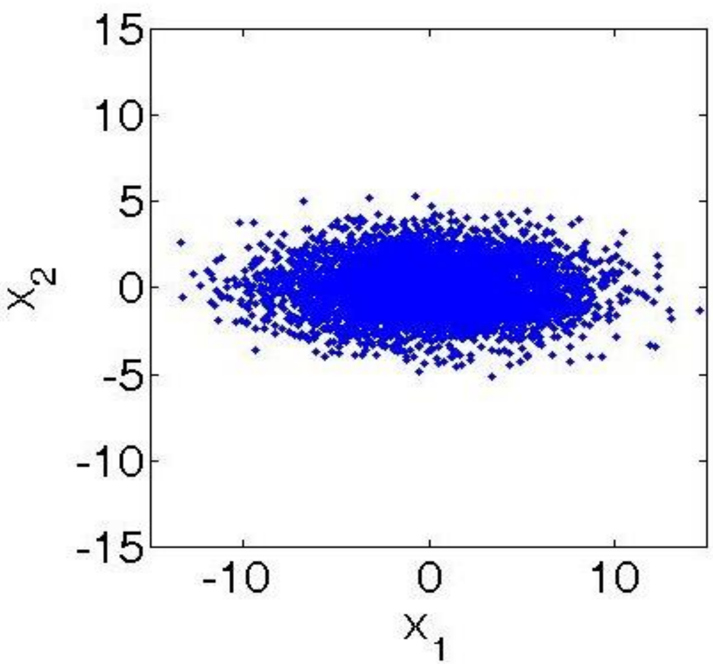
Sample Plot



Scatter Plot

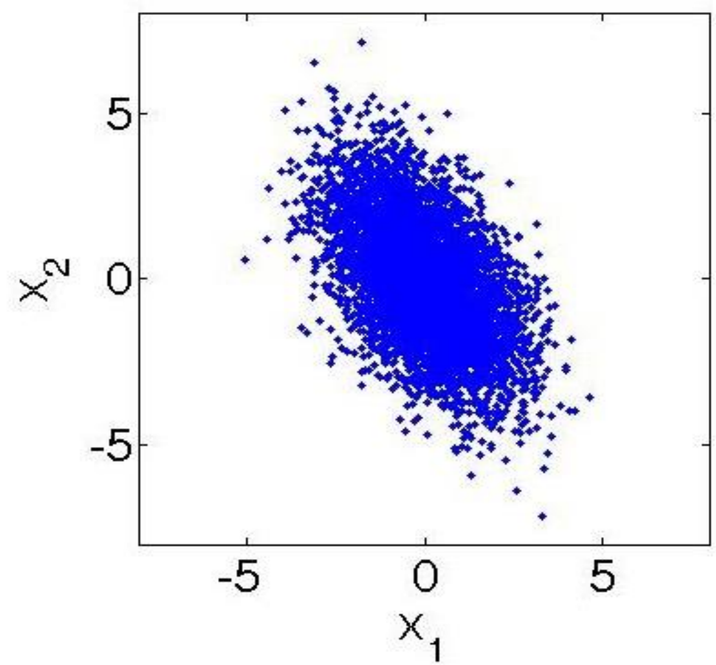
Linear Transforms of Gaussian Random Variables (continued)

$$\begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.75} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{s_1} \\ \mathbf{s_2} \end{bmatrix}$$



diagonal transforms, stretch the distribution along the axes

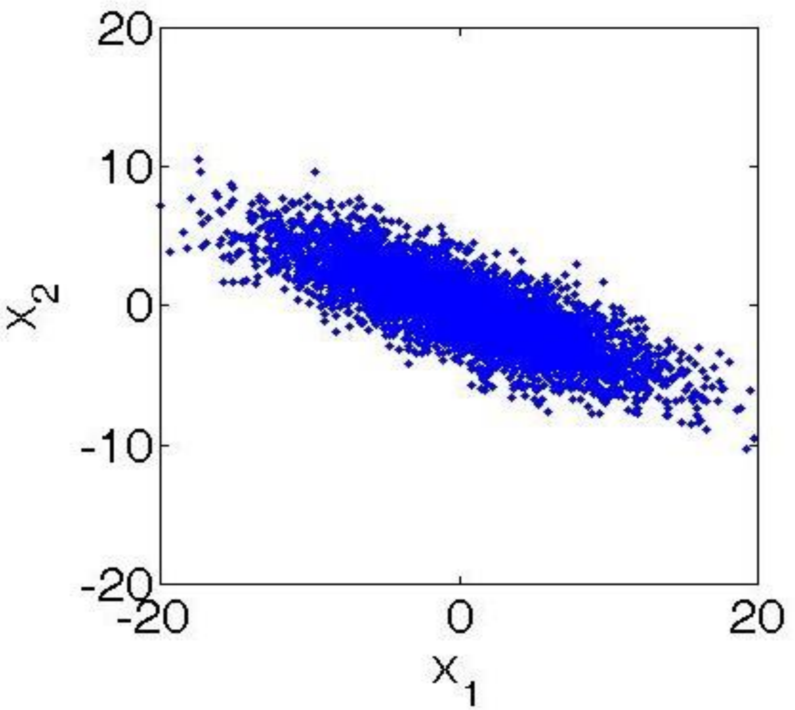
$$\begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \begin{bmatrix} \cos(\pi/6) & -\sin(\pi/6) \\ \sin(\pi/6) & \cos(\pi/6) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{s_1} \\ \mathbf{s_2} \end{bmatrix}$$



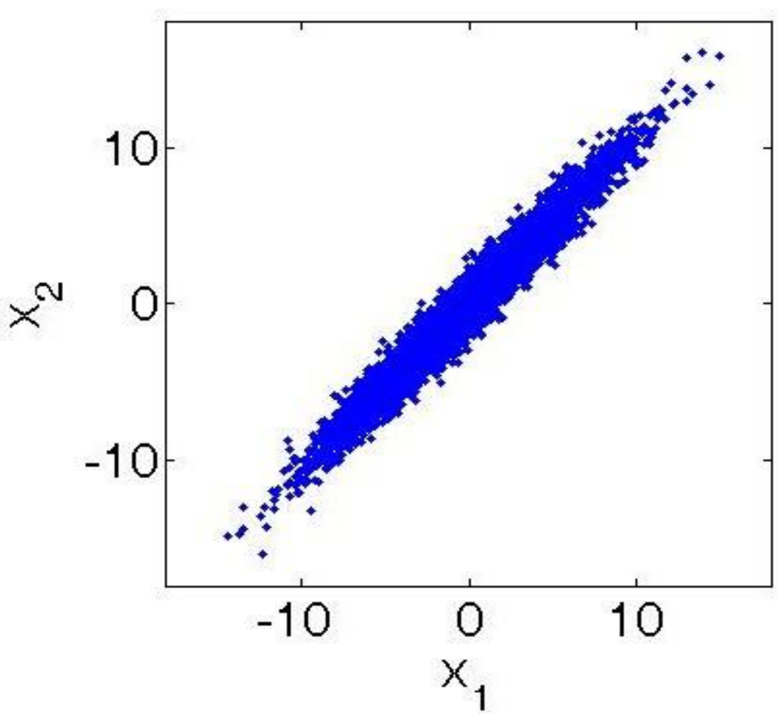
orthonormal transforms, rotate the distribution around the axes

Linear Transforms of Gaussian Random Variables (continued)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$



Conclusion: arbitrary transforms (rotation + scaling), rotate and stretch the distribution at the same time

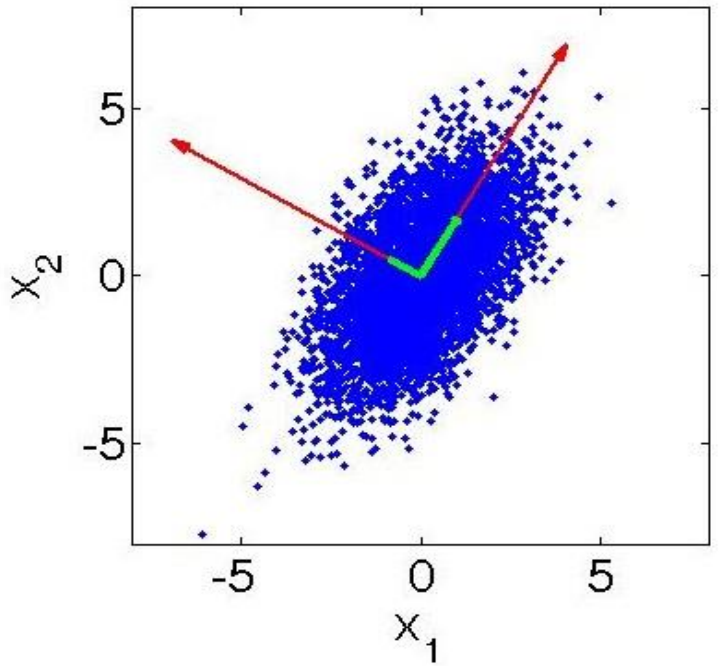
Eigenvalue Decomposition

For a zero-mean random vector **x**, the covariance matrix $\mathbf{C}_x=E\{\mathbf{x}\mathbf{x}^T\}$ can be decomposed as follows:

$$\mathbf{C}_x=\mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^T$$

where $\mathbf{U}=[\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N]$ is an orthonormal matrix with the eigenvectors of $\mathbf{C}_x$ in its columns, and $\boldsymbol{\Lambda}$ is a diagonal matrix of the eigenvalues of $\mathbf{C}_x$

2D example:



$$\mathbf{C}_x=\begin{bmatrix} E\{x_1^2\} & E\{x_1x_2\} \\ E\{x_2x_1\} & E\{x_2^2\} \end{bmatrix}$$

$$\mathbf{U}=[\mathbf{u}_1 \quad \mathbf{u}_2]$$

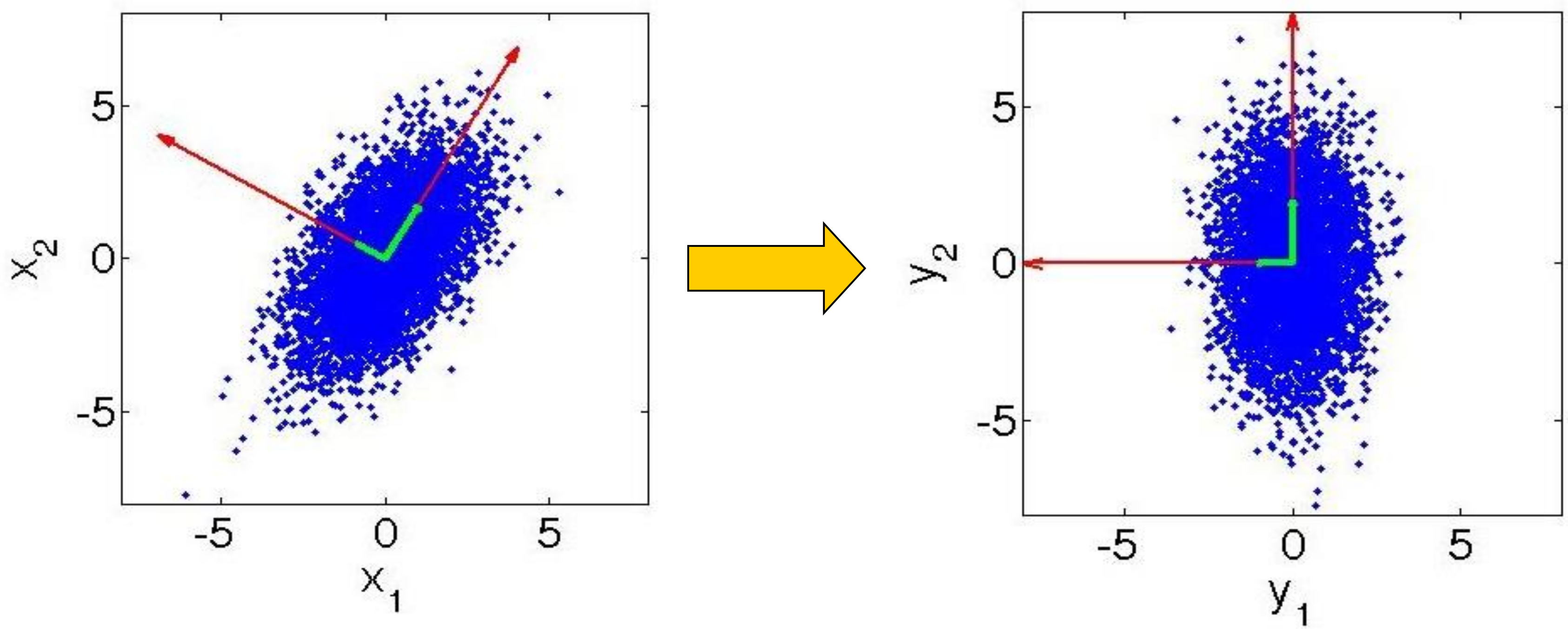
$$\boldsymbol{\Lambda}=\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Principal Component Analysis (PCA)

- Principal Component Analysis (PCA) also known as the (discrete) Karhunen-Loève transform or the Hotelling transform is a rotation transform that decorrelates the data (diagonalizes the covariance matrix):

$$\mathbf{Y} = \mathbf{U}^T\mathbf{X}$$

$$\mathbf{C}_Y = \mathbf{E}\{\mathbf{Y}\mathbf{Y}^T\} = \mathbf{E}\{\mathbf{U}^T\mathbf{X}\mathbf{X}^T\mathbf{U}\} = \mathbf{\Lambda}$$

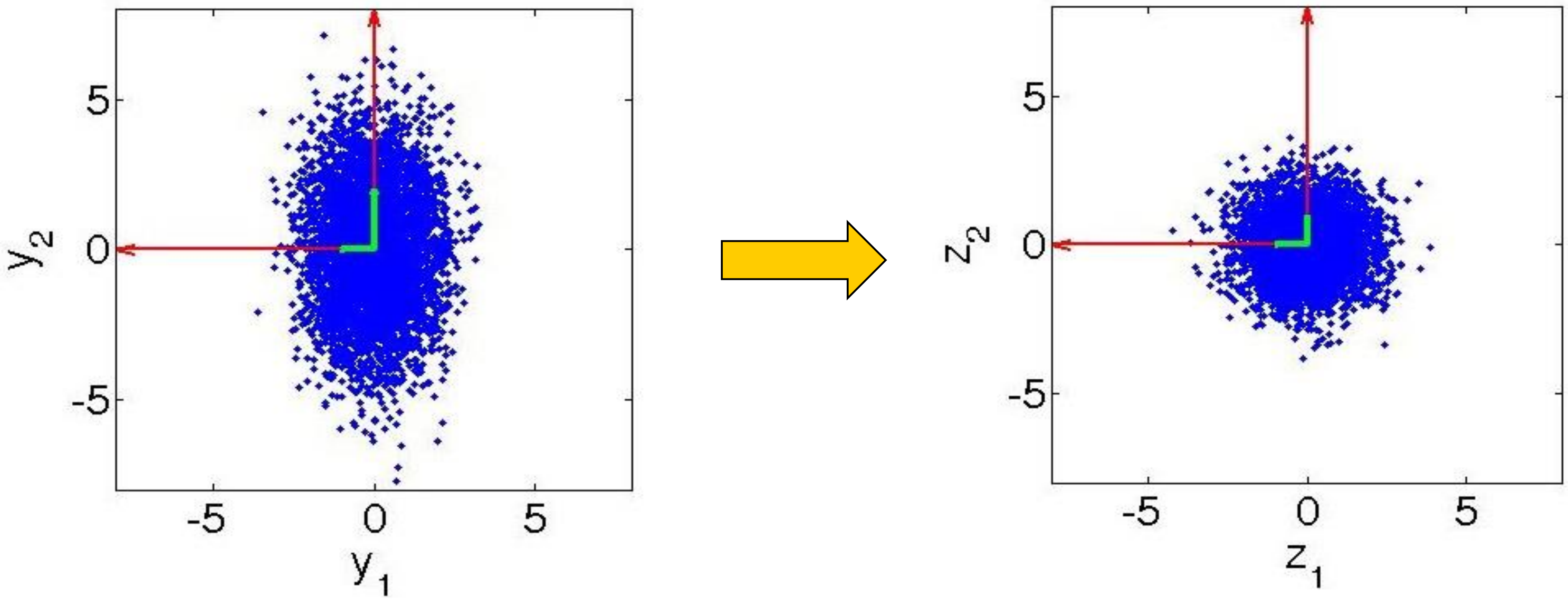


Note: Uncorrelatedness implies separation of variables up their second order statistics

Sphering

- We now go a step further and normalize the covariance matrix through the following transformation: $\mathbf{Z} = \mathbf{W.Y}$

$$\mathbf{C_Z} = \mathbf{E\{Z \cdot Z^T\}} = \mathbf{\Lambda^{-1/2} \cdot \Lambda \cdot \Lambda^{-1/2}} = \mathbf{I} \qquad \mathbf{W} = \mathbf{\Lambda^{-1/2}} = \begin{bmatrix} \sqrt{\boldsymbol{\lambda}_1} & \mathbf{0} \\ \mathbf{0} & \sqrt{\boldsymbol{\lambda}_2} \end{bmatrix}$$



Note: No further information in the second-order statistics; multiplication of **Z** by any othonormal (rotation) matrix preserves its whiteness:

$$\mathbf{Z'} = \mathbf{R.Z} \qquad \mathbf{C_{\underline{Z}}} = \mathbf{E\{Z' \cdot Z'^T\}} = \mathbf{R \cdot I \cdot R^T} = \mathbf{I}$$

Uncorrelatedness vs. Independence

- Uncorrelatedness of zero-mean RVs: $E\{x \cdot y\} = E\{x\} \cdot E\{y\}$

- Independence: $f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$

  $\rightarrow E\{g(x) \cdot h(y)\} = E\{g(x)\} \cdot E\{h(y)\}$

- Apparently, independence is far stronger than uncorrelatedness, as it relies on higher order statistics.
- For Gaussian random variables, uncorrelatedness is equivalent with independence. Therefore, their higher order statistics (HOS) do not contain any additional information.
- So, non-Gaussianity is the key point for conventional ICA algorithms, which use HOS for source separation

How Does Classical ICA Work?

Assumptions:

1. Data model (with equal number of source and sensors) : $\mathbf{X} = \mathbf{A} \cdot \mathbf{S}$
2. The sources are statistically independent
3. All (except at most one) of the sources are non-Gaussian

The Central Limit Theorem: The summation of N IID random variables, tends to Gaussian as N grows, regardless of their original distributions. So the summation of two random variables is "more Gaussian", than the original distribution: $\mathbf{y} = \mathbf{b}^{\mathbf{T}} \cdot \mathbf{X} = \mathbf{b}^{\mathbf{T}} \cdot \mathbf{A} \cdot \mathbf{S} = \mathbf{w}^{\mathbf{T}} \cdot \mathbf{S}$

y is more Gaussian than any of the components of **S**, except when only one of the elements of the vector **w** is nonzero.

Solution: we can seek "the most non-Gaussian" linear mixture of the measurements, which can be proved to be a linear copy of the original independent sources

How Does Classical ICA Work? (continued)

ICA algorithm overview:

1) Apply PCA and sphering to remove second order correlations (optional but very helpful stage)
2) Define a measure of Independence: Kurthosis (4^{th} Order Cummulant), Negentropy, Mutual Information, etc.
3) Maximize this measure over linear mixtures of the observed data, to find independent sources

Question: Is there any guarantee that independent components obtained from this algorithm are the originally mixed components?

Answer: Yes, to some extent!

Limitations of Classical ICA

- Requires the same number of sources as sensors
- Does not work for Gaussian data (at most one of the sources can be Gaussian)
- Can estimate the sources, up to a "linear copy":
  - Can't estimate the energy or sign of the signal:

$$\mathbf{X} = \mathbf{A} \cdot \mathbf{S} = (\alpha \mathbf{A}) \cdot (\alpha^{-1} \mathbf{S}) = \mathbf{A}' \cdot \mathbf{S}'$$

  where **α** is an arbitrary diagonal matrix or scalar

  - Can't estimate the order of the signals:

$$\mathbf{X} = \mathbf{A} \cdot \mathbf{S} = (\mathbf{AP}) \cdot \mathbf{P}^{-1} \mathbf{S} = \mathbf{A}' \cdot \mathbf{S}'$$

  where **P** is an arbitrary permutation matrix.

The FastICA Algorithm

1. Given **x**, pre-whiten the data by PCA and sphering to obtain **y**
2. Select a non-quadratic nonlinear function f(·), its first derivative g(·) and its second derivative g'(·) and an initial weight vector w. For example:

$$f(u) = \log\cosh(u), \quad g(u) = \tanh(u), \quad \text{and} \quad g'(u) = 1 - \tanh^2(u)$$

or

$$f(u) = -e^{-u^2/2}, \quad g(u) = ue^{-u^2/2}, \quad \text{and} \quad g'(u) = (1-u^2)e^{-u^2/2}$$

3. Let $\mathbf{w}^+ \leftarrow E\{\mathbf{y}g(\mathbf{w}^T\mathbf{y})^T\} - E\{g'(\mathbf{w}^T\mathbf{y})\}\mathbf{w}$, where E{·} denotes averaging over all column-vectors of the matrix **y**
4. Let $\mathbf{w} \leftarrow \mathbf{w}^+/\|\mathbf{w}\|$
5. If not converged go to step 3

Problem Restatement: Degrees of Freedom in Linear Algebraic Transforms

We start from a totally different viewpoint:

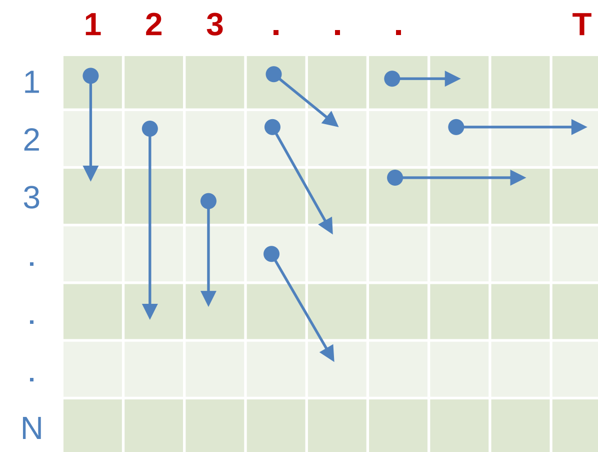
In a linear transform $\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t)$ where

- $\mathbf{x}(t) \in R^{N\times T}$: observed data
- $\mathbf{y}(t) \in R^{N\times T}$: transformed data with desired set of properties
- $\mathbf{W} \in R^{N\times N}$: unknown separating matrix

The problem of linear source separation can be viewed as a matrix design problem with N^2 degrees of freedom (DoF), to obtain $\mathbf{y}(t)$ with desired properties

Covariance Matrices and Tensor Slices

- Consider N×N matrices representing different statistics between different channels of $\mathbf{x}(t)$:
  - $\mathbf{C} = E\{\mathbf{x}(t)\mathbf{x}^T(t)\}$: The covariance matrix
  - $\mathbf{C}(\tau) = E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau)\}$: The cross-correlation matrix
  - $\mathbf{C}_0 = E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau)x_k(t+\tau_0)\}$: Slices of higher-order statistics tensors in different time-lags
  - $\mathbf{C}_1 = E\{\mathbf{x}(t)\mathbf{x}^T(t)x_k(t)\}$: Slices of tensors



different possible correlation directions

- These matrices carry average inter-channel similarities (correlation) over time
- Diagonalization of a number or all of these matrices results in the decorrelation and eventually independence of the corresponding channels

Covariance Matrices and Tensor Slices

Example: Consider $\mathbf{x}(t)$ with covariance matrix $\mathbf{C}_0=E\{\mathbf{x}(t)\mathbf{x}^T(t)\}$ and lagged-covariance matrices $\mathbf{C}_1=E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau_1)\}$, $\mathbf{C}_2=E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau_2)\}$, …, $\mathbf{C}_k=E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau_k)\}$

- For $\mathbf{y}(t)=\mathbf{W}\mathbf{x}(t)$:

$$\mathbf{A}_0 = E\{\mathbf{y}(t)\mathbf{y}^T(t)\} = E\{\mathbf{W}\mathbf{x}(t)\mathbf{x}^T(t)\mathbf{W}^T\} = \mathbf{W}E\{\mathbf{x}(t)\mathbf{x}^T(t)\}\mathbf{W}^T= \mathbf{W}\mathbf{C}_0\mathbf{W}^T$$

$$\cdots$$

$$\mathbf{A}_k = E\{\mathbf{y}(t)\mathbf{y}^T(t+\tau_k)\} = E\{\mathbf{W}\mathbf{x}(t)\mathbf{x}^T(t+\tau_k)\mathbf{W}^T\} = \mathbf{W}E\{\mathbf{x}(t)\mathbf{x}^T(t+\tau_k)\}\mathbf{W}^T= \mathbf{W}\mathbf{C}_k\mathbf{W}^T$$

- These matrices show the inter-channel correlations of the output $\mathbf{y}(t)$, at different time lags.
- For temporally correlated data, diagonalizing all such matrices results in channel independence

Question: Can we find a linear transformation $\mathbf{y}(t)=\mathbf{W}\mathbf{x}(t)$, which simultaneously diagonalizes all or a number of the above matrices?

Covariance Matrices and Tensor Slices (continued)

Some facts from linear algebra:

1- Eigenvalue Decomposition: a symmetric matrix $\mathbf{C}_0$ can be diagonalized as follows:

$$\mathbf{W}\mathbf{C}_0\mathbf{W}^T = \mathbf{\Lambda}$$

- $\mathbf{C}_0$ is a symmetric matrix with N(N+1)/2 DoF
- $\mathbf{\Lambda}$ is a diagonal matrix with N DoF
- **W** is the orthonormal eigen-matrix with N(N-1)/2 DoF

Therefore

$$\mathrm{DoF}(\mathbf{C}_0) = \mathrm{DoF}(\mathbf{\Lambda}) + \mathrm{DoF}(\mathbf{W})$$

Covariance Matrices and Tensor Slices (continued)

2- In Generalized Eigenvalue Decomposition: two symmetric matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ can be diagonalized as follows:

$$\mathbf{W}\mathbf{C}_0\mathbf{W}^T = \mathbf{I}$$

$$\mathbf{W}\mathbf{C}_1\mathbf{W}^T = \mathbf{\Lambda}$$

- $\mathbf{C}_0$ and $\mathbf{C}_1$ are symmetric matrices, each with N(N+1)/2 DoF
- $\mathbf{\Lambda}$ is a diagonal matrix with N DoF
- $\mathbf{I}$ is the identity matrix with no DoF
- $\mathbf{W}$ is a square matrix with N^2 DoF

Therefore

$$\mathrm{DoF}(\mathbf{C}_0) + \mathrm{DoF}(\mathbf{C}_1) = \mathrm{DoF}(\mathbf{\Lambda}) + \mathrm{DoF}(\mathbf{W})$$

Covariance Matrices and Tensor Slices

(continued)

3- Simultaneous diagonalization of multiple symmetric matrices: No more than two symmetric matrices can be simultaneously diagonalized, except if they belong to the same eigen-space:

$$\mathbf{W}\mathbf{C}_0\mathbf{W}^T = \mathbf{I}$$

$$\mathbf{W}\mathbf{C}_1\mathbf{W}^T = \boldsymbol{\Lambda}_1$$

$$\dots$$

$$\mathbf{W}\mathbf{C}_k\mathbf{W}^T = \boldsymbol{\Lambda}_{\mathbf{k}}$$

Source Separation by Matrix Diagonalization

Based on these facts, we can consider two classes of source separation methods:

- Exact diagonalization of two well-chosen matrices

Advantage: The corresponding statistics are fully separated

Disadvantage: Requires prior information for selecting the matrices

Major methods of this class: AMUSE, ПCA, etc.

- Approximate diagonalization of more than two matrices

Advantage: It is easier to select the matrices

Disadvantage: The corresponding statistics are only separated approximately; measures of approximate diagonalization are not unique

Major methods of this class: JADE, SOBI, etc.

Fetal ECG Extraction using Multichannel Recordings

Application of Blind and Semi-Blind Source Separation for Fetal ECG Extraction

Multichannel Maternal Abdominal Signal Model

Due to the physical properties of the maternal and fetal heart presented in the introduction, the maternal abdominal recordings can be considered as follows:

$$\mathbf{x}(\mathrm{t}) = \mathbf{H}_{\mathrm{m}}\mathbf{s}_{\mathrm{m}}(\mathrm{t}) + \mathbf{H}_{\mathrm{f}}\mathbf{s}_{\mathrm{f}}(\mathrm{t}) + \mathbf{n}(\mathrm{t})$$

where

- $\mathbf{x}(\mathrm{t}) \in \mathbb{R}^N$ is N channels of maternal abdominal recordings at time t
- $\mathbf{n}(\mathrm{t}) \in \mathbb{R}^N$ is the observation noise
- $\mathbf{s}_{\mathrm{m}}(\mathrm{t}) \in \mathbb{R}^M$ is the maternal ECG components with M effective dimensions
- $\mathbf{s}_{\mathrm{f}}(\mathrm{t}) \in \mathbb{R}^L$ is the fetal ECG components with L effective dimensions
- $\mathbf{H}_{\mathrm{m}} \in \mathbb{R}^{N\times M}$ and $\mathbf{H}_{\mathrm{f}} \in \mathbb{R}^{N\times L}$ are unknown linear matrices, which map the heart signals to the maternal abdomen
- $\mathbf{s}_{\mathrm{m}}(\mathrm{t})$ and $\mathbf{s}_{\mathrm{f}}(\mathrm{t})$ are statistically independent

Note:

- This data model is very accurate in stationary scenarios (when $\mathbf{H}_{\mathrm{m}}$ and $\mathbf{H}_{\mathrm{f}}$ are constant matrices)
- In theory, due to the spatial distributed-ness of the heart source, M and L tend to infinity. However in practice, using far field approximations, the effective number of dimensions are $M = 5 \dots 6$ and $L = 2 \dots 3$.

Ref: Sameni, R., Jutten, C., & Shamsollahi, M. B. What ICA provides for ECG processing: Application to noninvasive fetal ECG extraction. *IEEE International Symposium on Signal Processing and Information Technology, Aug. 2006* (pp. 656-661).

A BSS-Based Formulation of Multichannel Maternal Abdominal Signals

- The previous data model can be rewritten in terms of a BSS problem:

$$\mathbf{x}(\mathrm{t}) = [\mathbf{H}_\mathbf{m} \quad | \quad \mathbf{H}_\mathrm{f}] \begin{bmatrix} \mathbf{s}_\mathrm{m}(\mathrm{t}) \\ - \\ \mathbf{s}_\mathrm{f}(\mathrm{t}) \end{bmatrix} + \mathbf{n}(\mathrm{t})$$

$$= \mathbf{Hs}(\mathrm{t}) + \mathbf{n}(\mathrm{t})$$

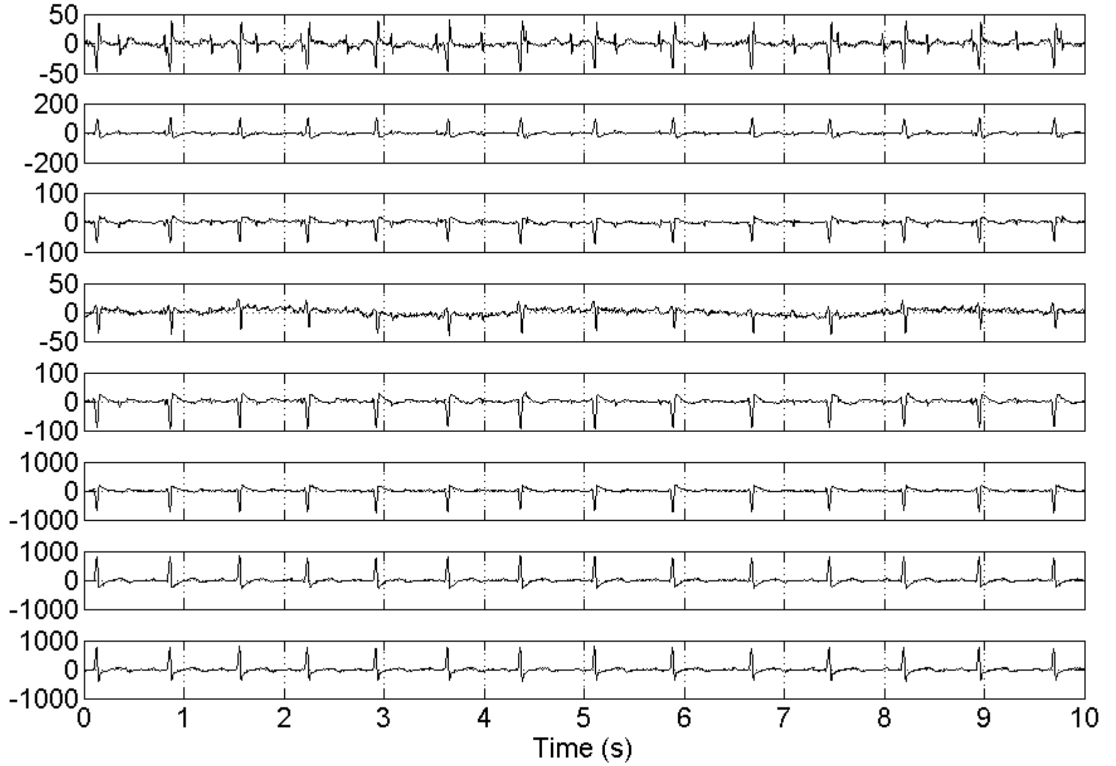
  which is in the form of a linear instantaneous noisy BSS problem.
- In this formulation, if the abdominal channels are well-spread across the maternal abdomen (to avoid singular mixtures) and if $M + L \leq N$ (the sum of maternal and fetal effective number of dimensions is equal or less than the number of recordings), the BSS problem is determined or over-determined.
- ICA (or more precisely, independent subspace analysis: ISA) can be used to separate the maternal and fetal ECG signals

- That's why fECG extraction has been one of the favorite problems for BSS experts!

Further reading on ISA: J.-F. Cardoso, "Multidimensional independent component analysis," in Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '98), vol. 4, May 1998, pp. 1941– 1944.

Naïve ICA for Fetal ECG Extraction

- With a good electrode placement, even the most classical ICA methods are quite successful in fECG extraction from maternal abdominal recordings.
- Example: The DaISy fECG dataset



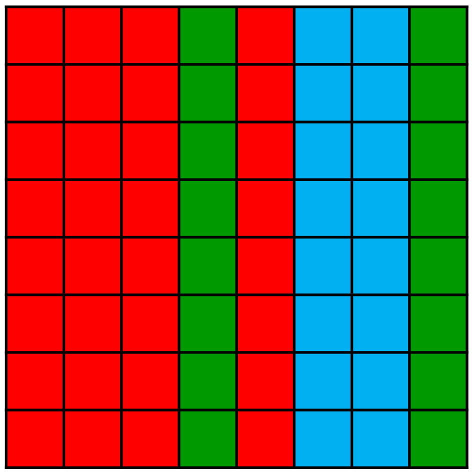
Refer to:

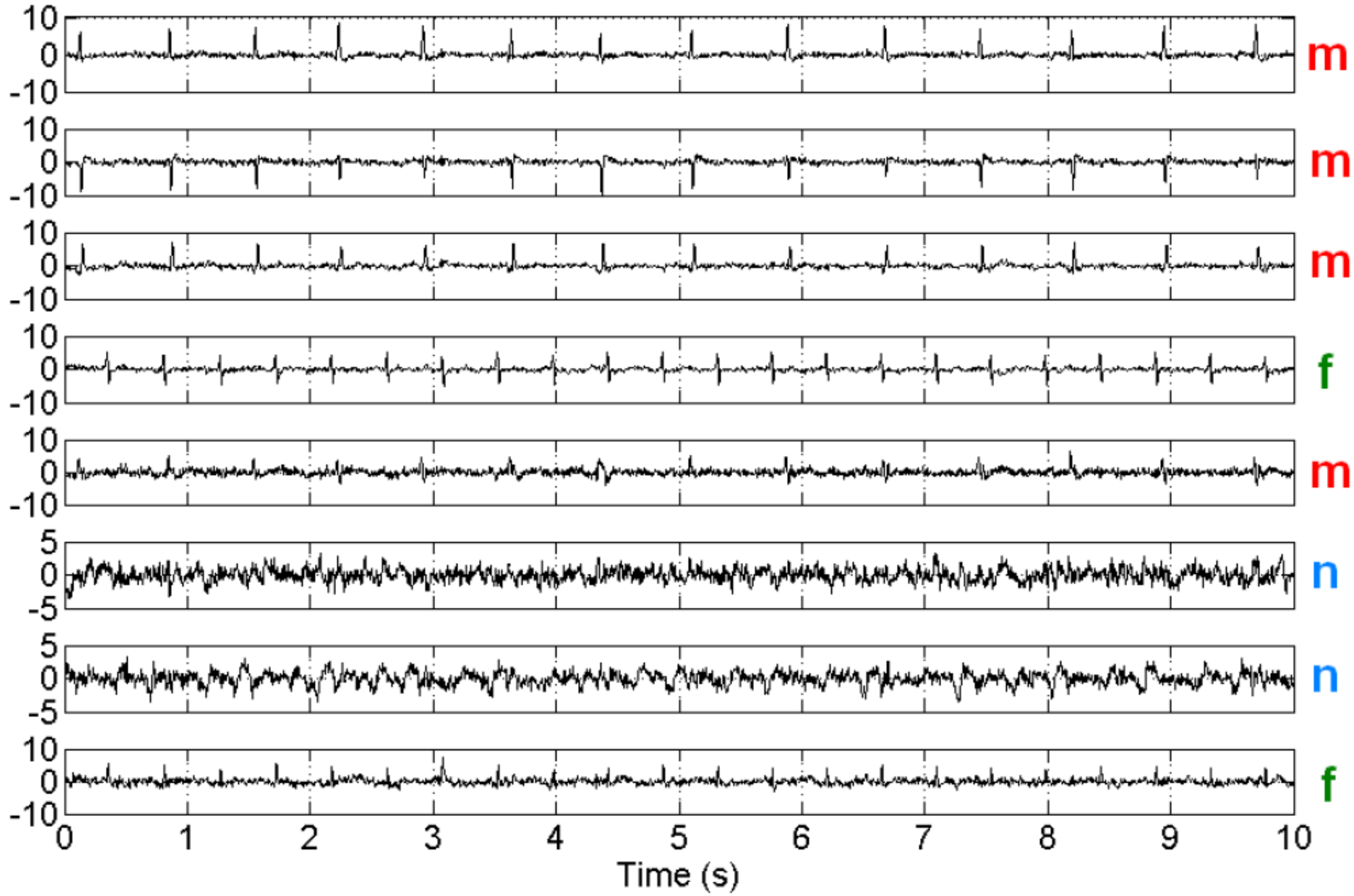
- L. De Lathauwer, B. De Moor, and J. Vandewalle, Fetal electrocardiogram extraction by blind source subspace separation," IEEE Trans. Biomed. Eng., vol. 47, pp. 567-572, May 2000.
- Zarzoso, V., & Nandi, A. K. (2001). Noninvasive fetal electrocardiogram extraction: blind separation versus adaptive noise cancellation. IEEE Transactions on biomedical engineering, 48(1), 12-18.

Naïve ICA for Fetal ECG Extraction (continued)

Independent components extracted by the JADE algorithm

Mixing matrix (**A**)
maternal/fetal/noise subspace
partitioning





Note: Quite similar results are obtained from other ICA methods such as FastICA, SOBI, TDSEP, etc. on this dataset.

Issues of Classical ICA for fECG Extraction

- Intrinsic limitations of ICA:
  - Source orders can not be determined
  - Source amplitude and sign can not be recovered
- Classical ICA assumptions do not exactly hold in this problem:
  - Number of sources and sensors are not equal
  - The mixtures can become singular due to electrode placement or fetal position
  - The mixture is always noisy; no guarantee for source extraction
  - Time-varying mixtures, due to fetal motion

Special source separation techniques have been proposed in the literature to resolve these issues

Semi-Blind Source Separation

- While classical ICA are only based on the assumption of statistical independence of the sources and linear time-invariance of the model, additional problem-specific priors can be used to tailor source separation algorithms for fECG extraction
- Such domain-specific techniques are known as semi-blind source separation and are (commonly) more powerful than generic ICA methods
- Prior assumptions regarding fetal/maternal ECG mixtures:
  1. Pseudo-periodicity of the maternal and fetal ECG
  2. Synchrony of the R-peaks (regardless of the morphologic variations of the leads)
  3. Inter-group independence and intra-group dependence between the maternal and fetal ECG components (groups of independent subspaces)

Separation of Periodic Sources

- Objective: To find a special linear transform $\mathbf{y}(\mathrm{t}) = \mathbf{B}\mathbf{x}(\mathrm{t})$ for decomposing multichannel signals into periodic components. The components should be ranked according to their relevance

- Method: Gather measures of signal periodicity in $\mathbf{C}_1$ and $\mathbf{C}_2$, and find a $\mathbf{B}$ to diagonalize them:

$$\mathbf{B}\mathbf{C}_1\mathbf{B}^{\top} = \mathbf{I}\ ,\ \mathbf{B}\mathbf{C}_2\mathbf{B}^{\top} = \mathbf{\Lambda}$$

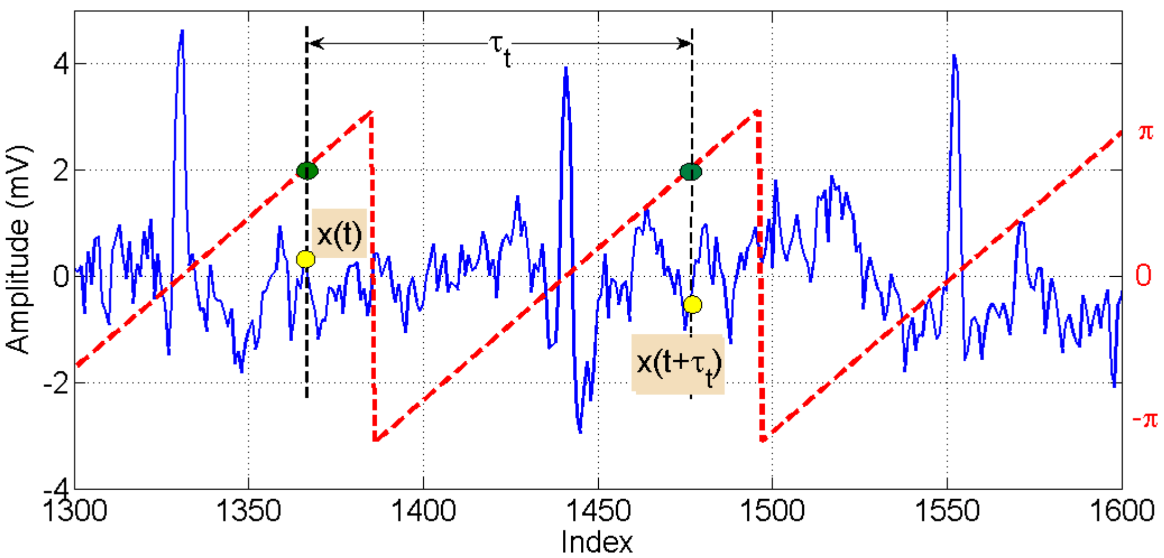
- Solution: Generalized Eigenvalue Decomposition (GEVD) of $(\mathbf{C}_1,\mathbf{C}_2)$
- The method is related to algebraic ICA methods, such as AMUSE and SOBI
- Question: How to choose the time-lags? How to handle pseudo-periodic signals such as the ECG?

AMUSE: Tong, L., Soon, V. C., Huang, Y. F., & Liu, R. A. L. R. (1990, May). AMUSE: a new blind identification algorithm. In *Circuits and Systems, 1990., IEEE International Symposium on* (pp. 1784-1787). IEEE.

Periodic Component Analysis (πCA) Algorithm

The modified ΠCA algorithm for ECG extraction:

1. Detect the R-peaks of the ECG of interest (a priori information)
2. Calculate the ECG phase signal $\theta(t)$
3. Calculate the time-varying lag: $\tau_t = \mathrm{argmin}\{\tau |\ \theta(t+\tau) = \theta(t), \tau>0\}$
4. Calculate $\mathbf{C}_x = Et\{\mathbf{x}(t)\mathbf{x}(t)^T\}$ and $\hat{\mathbf{C}}_x = E_t\{\mathbf{x}(t+\tau_t)\mathbf{x}(t)^T\}$
5. Symmetrize the lagged covariance matrix $\hat{\mathbf{C}}_x \leftarrow [\hat{\mathbf{C}}_x + \hat{\mathbf{C}}_x^T]/2$
6. Find $\mathbf{B} = \mathrm{GEVD}(\mathbf{C}_x, \hat{\mathbf{C}}_x)$
7. Calculate $\mathbf{y}(t) = \mathbf{B}\mathbf{x}(t)$

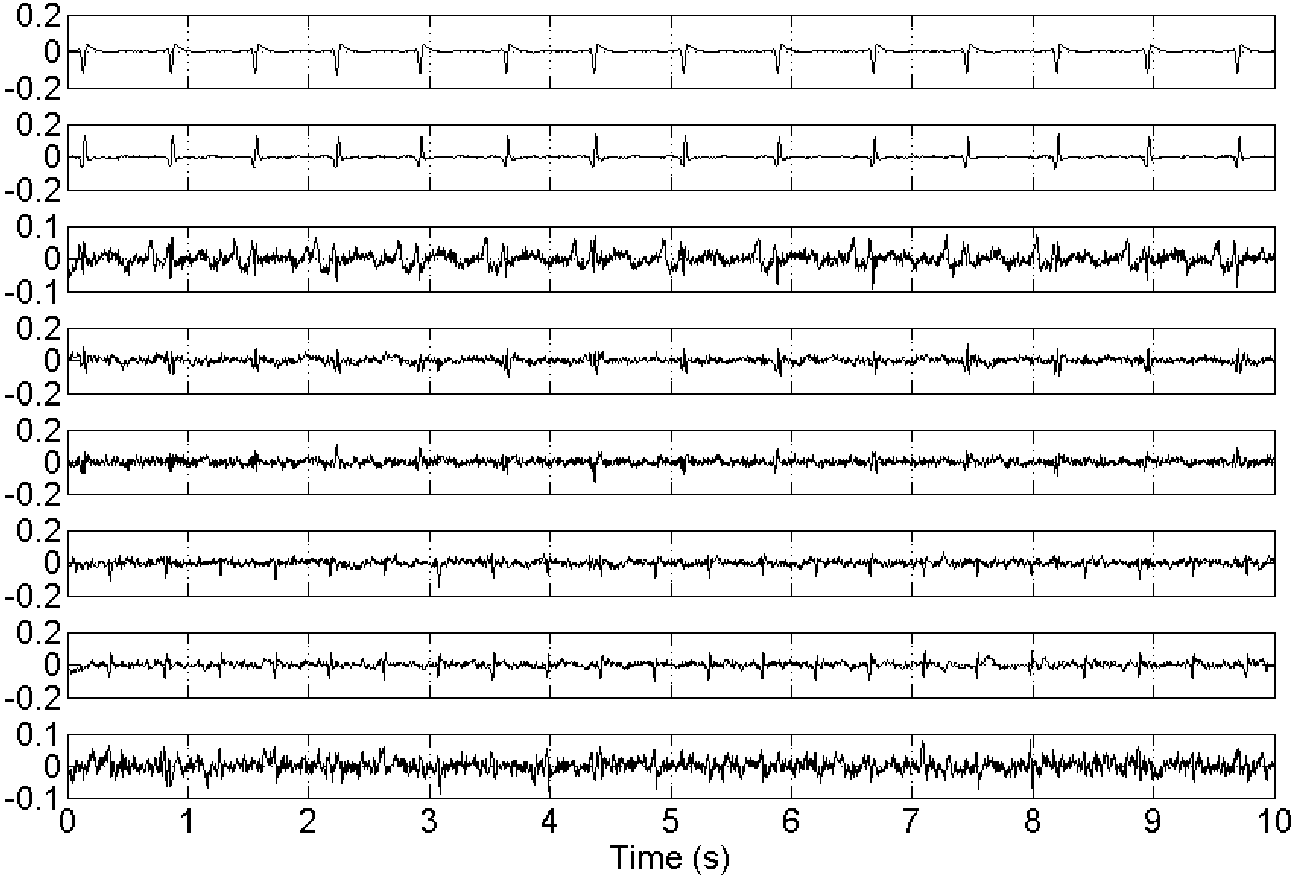


Ref:

- L. K. Saul and J. B. Allen, Periodic component analysis: An eigenvalue method for representing periodic structure in speech." in NIPS, 2000, pp. 807-813
- R. Sameni, C. Jutten, MB Shamsollahi (2008). Multichannel electrocardiogram decomposition using periodic component analysis. IEEE Transactions on Biomedical Engineering, 55(8), 1935-1940.

Periodic Component Analysis (πCA)

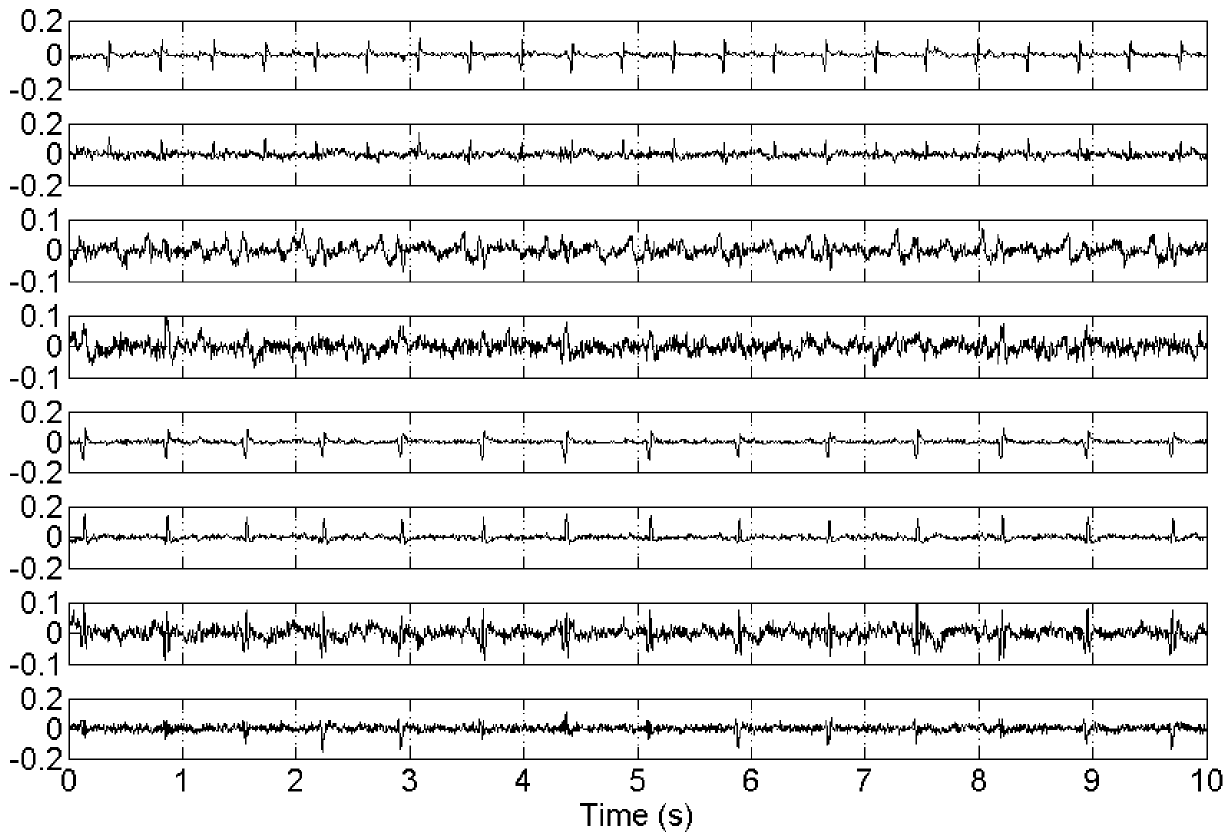
Example: πCA using maternal R-peaks



Property: sorted according to similarity with maternal ECG from top to bottom

Periodic Component Analysis (πCA)

Example: πCA using fetal R-peaks (from scalp leads or after an initial ICA or maternal-based πCA stage)



Property: sorted according to similarity with fetal ECG from top to bottom

Noisy and Singular Measurements

- Looking back at the data model $\mathbf{x(t)=Hs(t)+n(t)}$, we have: $\mathbf{y(t)=Bx(t)= BHs(t)+Bn(t)}$.
- Accordingly, $\mathbf{y}(\mathrm{t})$ **is only** an exact estimate of $\mathbf{s}(\mathrm{t})$, when:

1. $\mathbf{B}$ is the inverse of $\mathbf{H}$; no solutions when $\mathbf{H}$ is singular!

2. $\mathbf{Bn(t)=0}$ ($\mathbf{n(t)}$ is in the null-space of $\mathbf{B}$); otherwise the noise remains (or is even magnified) and the estimation of $\mathbf{B}$ is biased

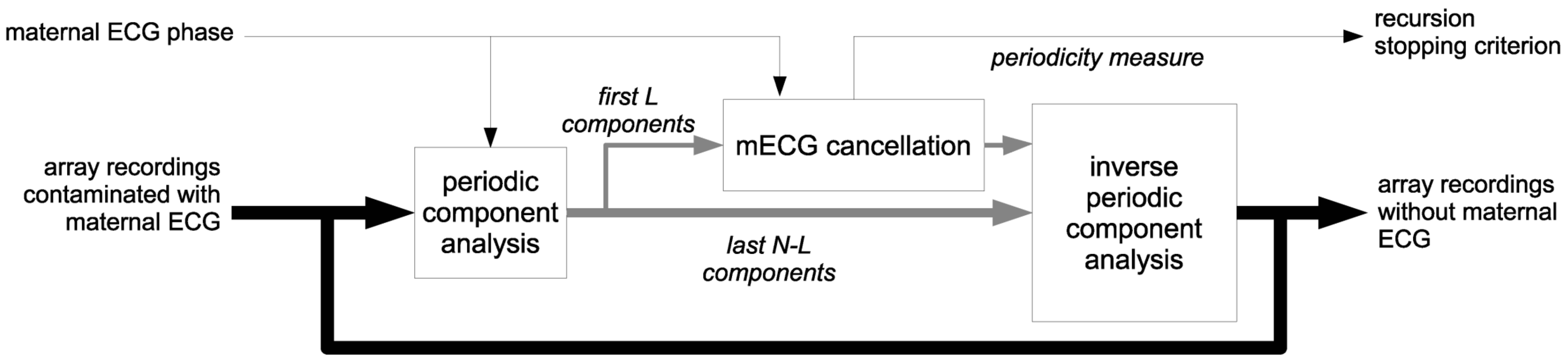
Solutions:

- Breaking the linearity of the transform!
- Denoisning before source separation

The two issues can be considered in a single algorithm, which performs source separation and denoising in a deflation-wise (step-by-step) manner.

A Deflation Procedure for fECG Extraction

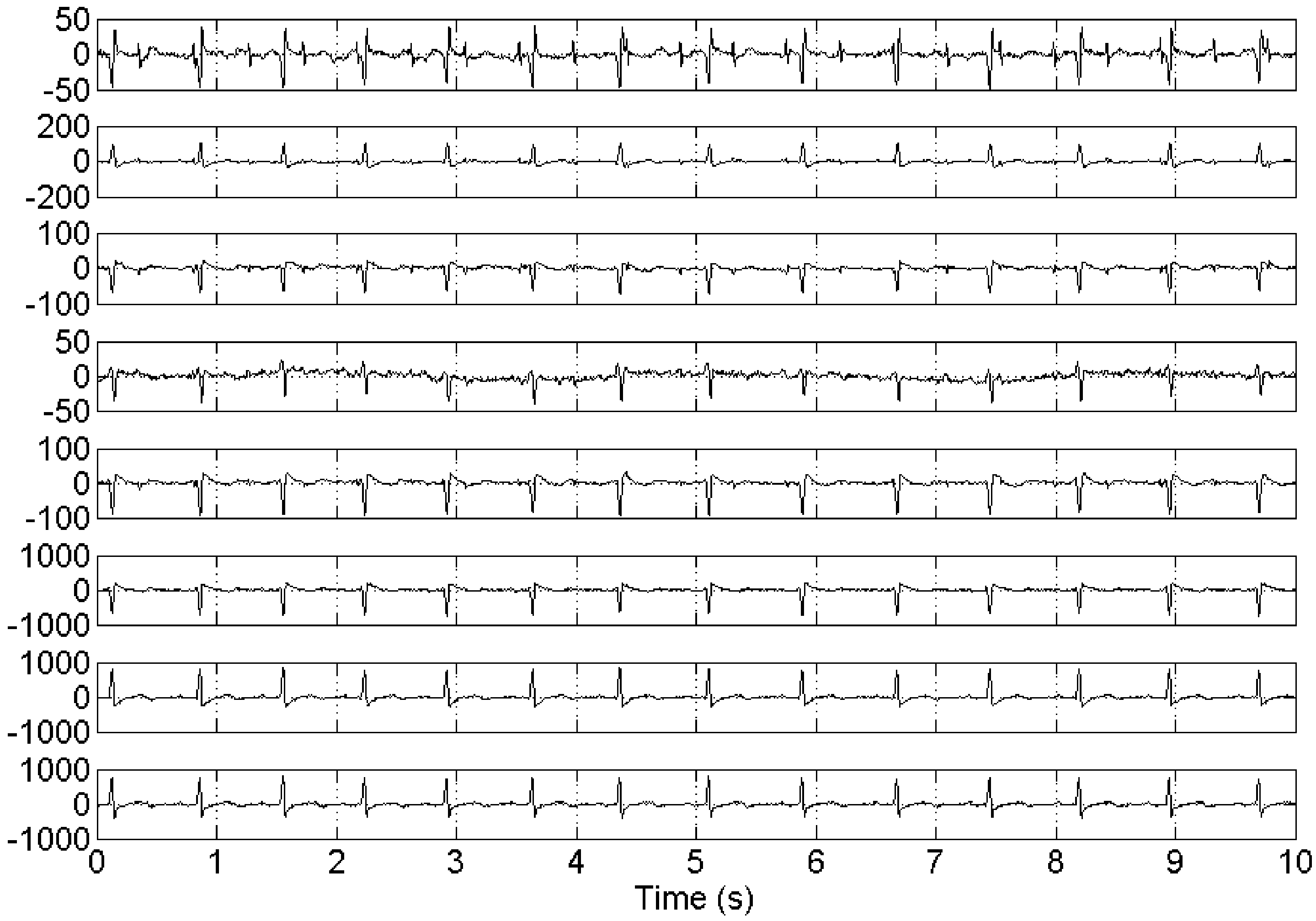
- The following iterative procedure can solve the problem of mixture singularity and noisy measurements at the same time:



Refs:
•R. Sameni, C. Jutten, MB Shamsollahi (2010). A deflation procedure for subspace decomposition. *IEEE Transactions on Signal Processing*, *58*(4), 2363-2374.
•R. Sameni, C. Jutten, M. Shamsollahi, and G. Clifford, “Extraction of Fetal Cardiac Signals,” U.S. Patent US 2010/0137727 A1, June 3, 2010.

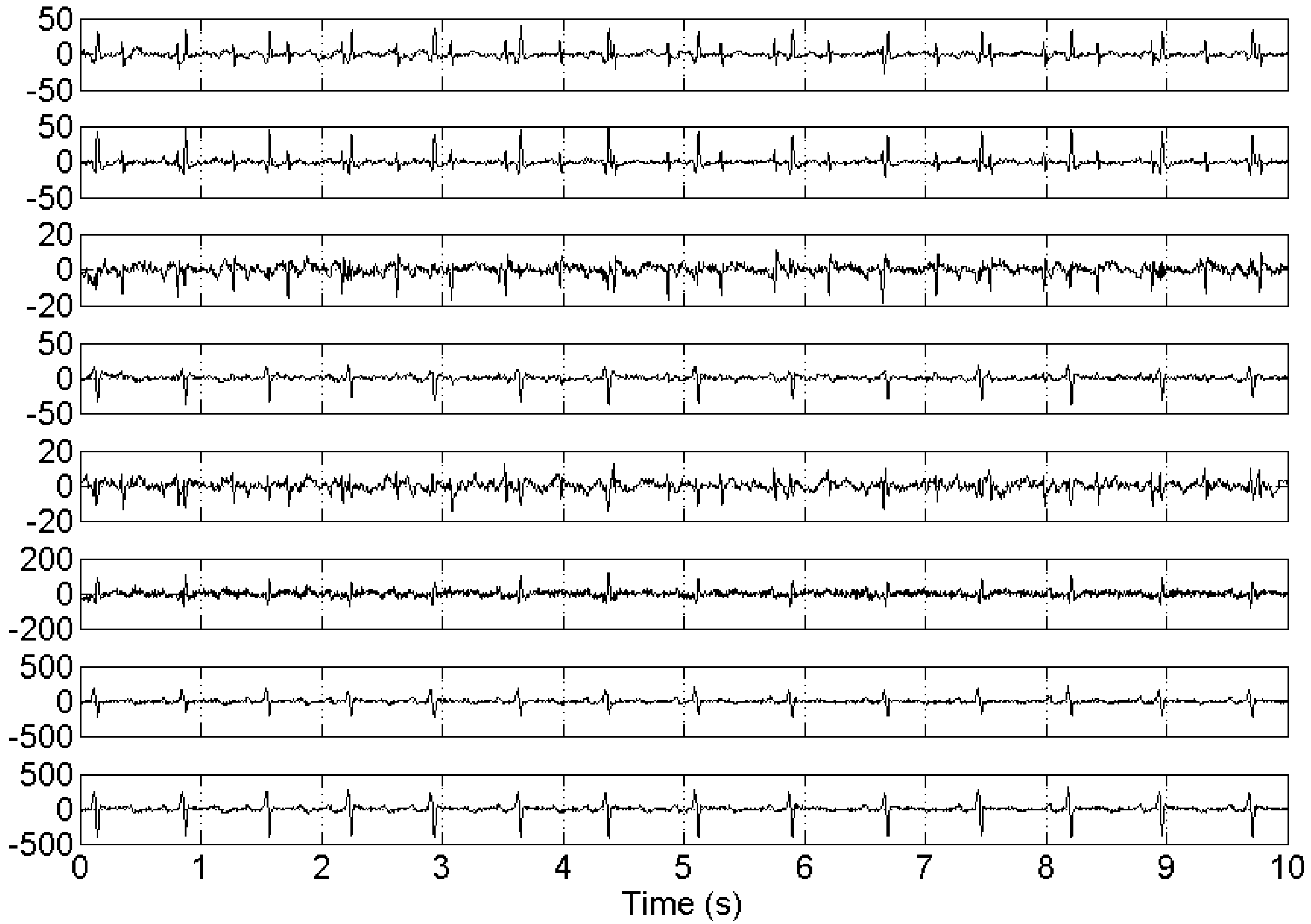
A Deflation Procedure for fECG Extraction
(continued)

Example 1: Original signal



A Deflation Procedure for fECG Extraction
(continued)

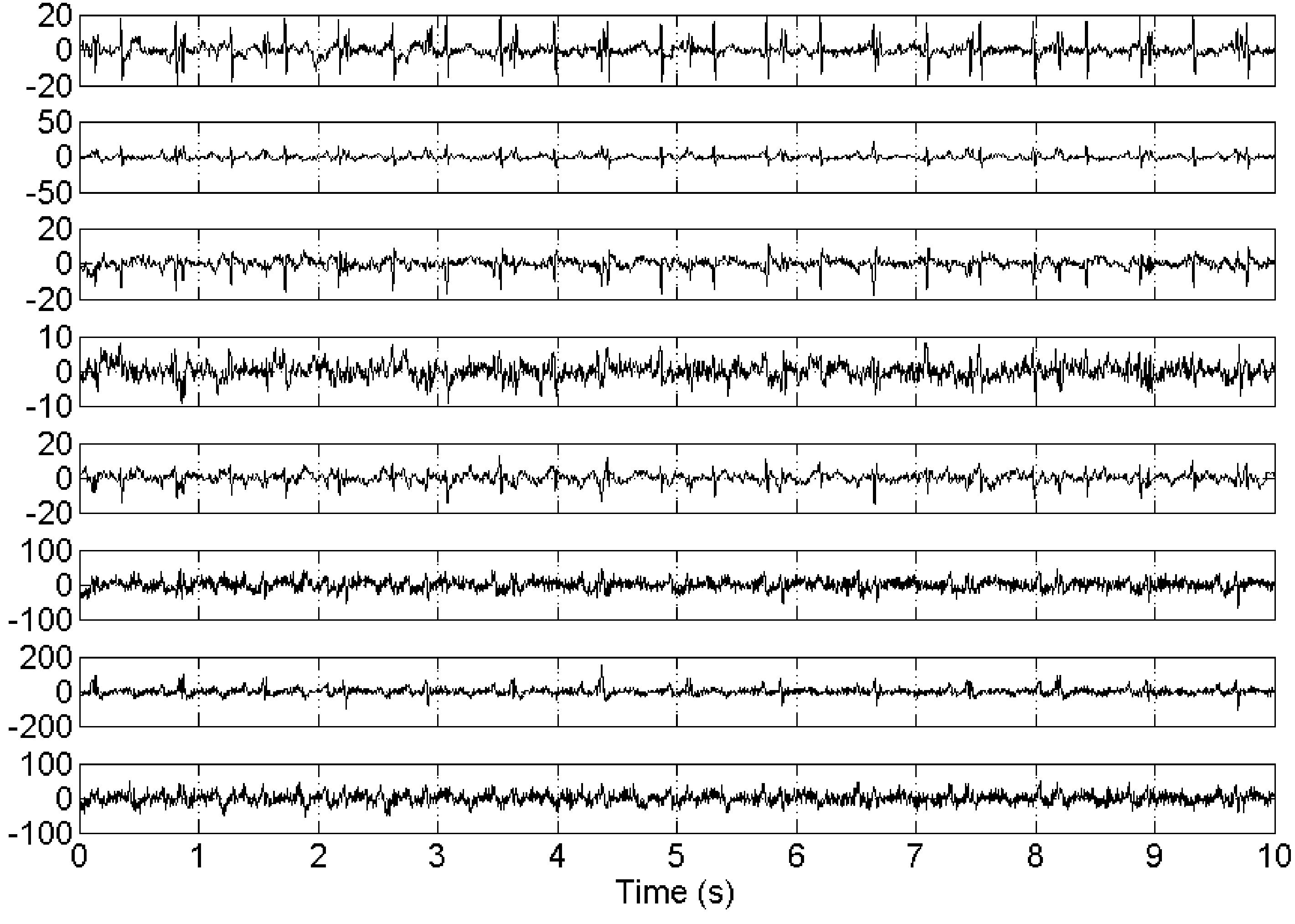
Example 1: After 1st iteration



A Deflation Procedure for fECG Extraction

(continued)

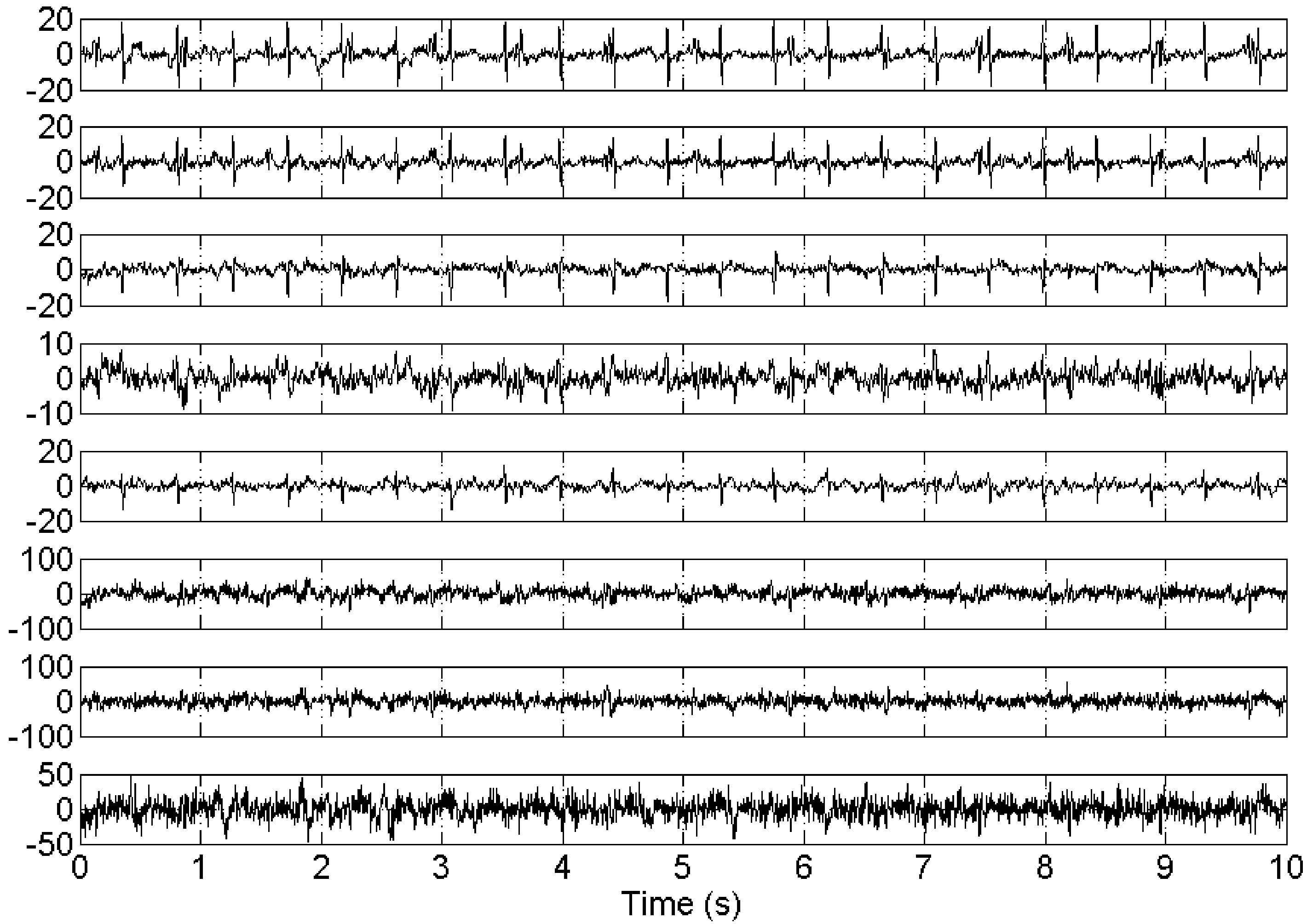
Example 1: After 2nd iteration



A Deflation Procedure for fECG Extraction

(continued)

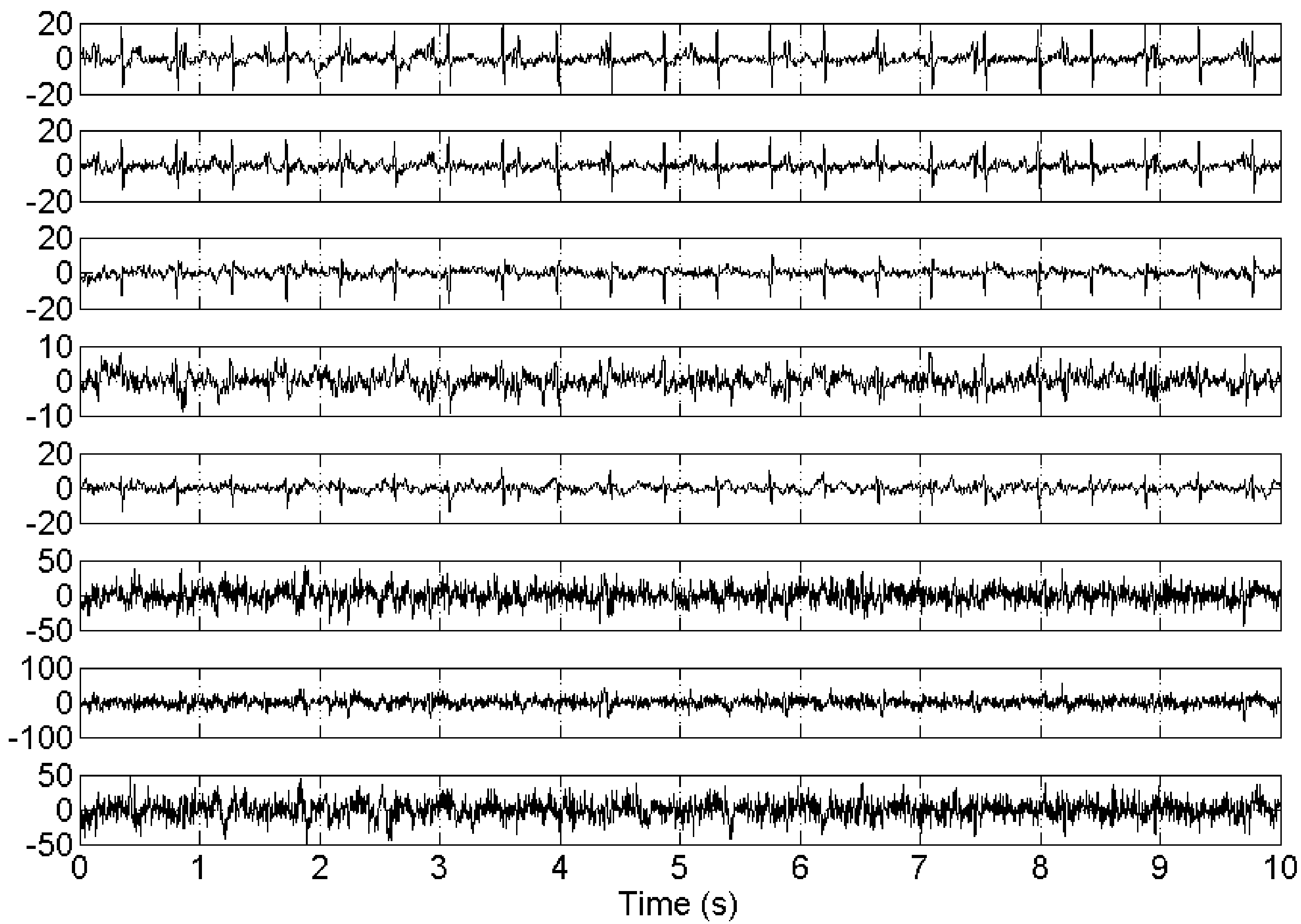
Example 1: After 3rd iteration



A Deflation Procedure for fECG Extraction

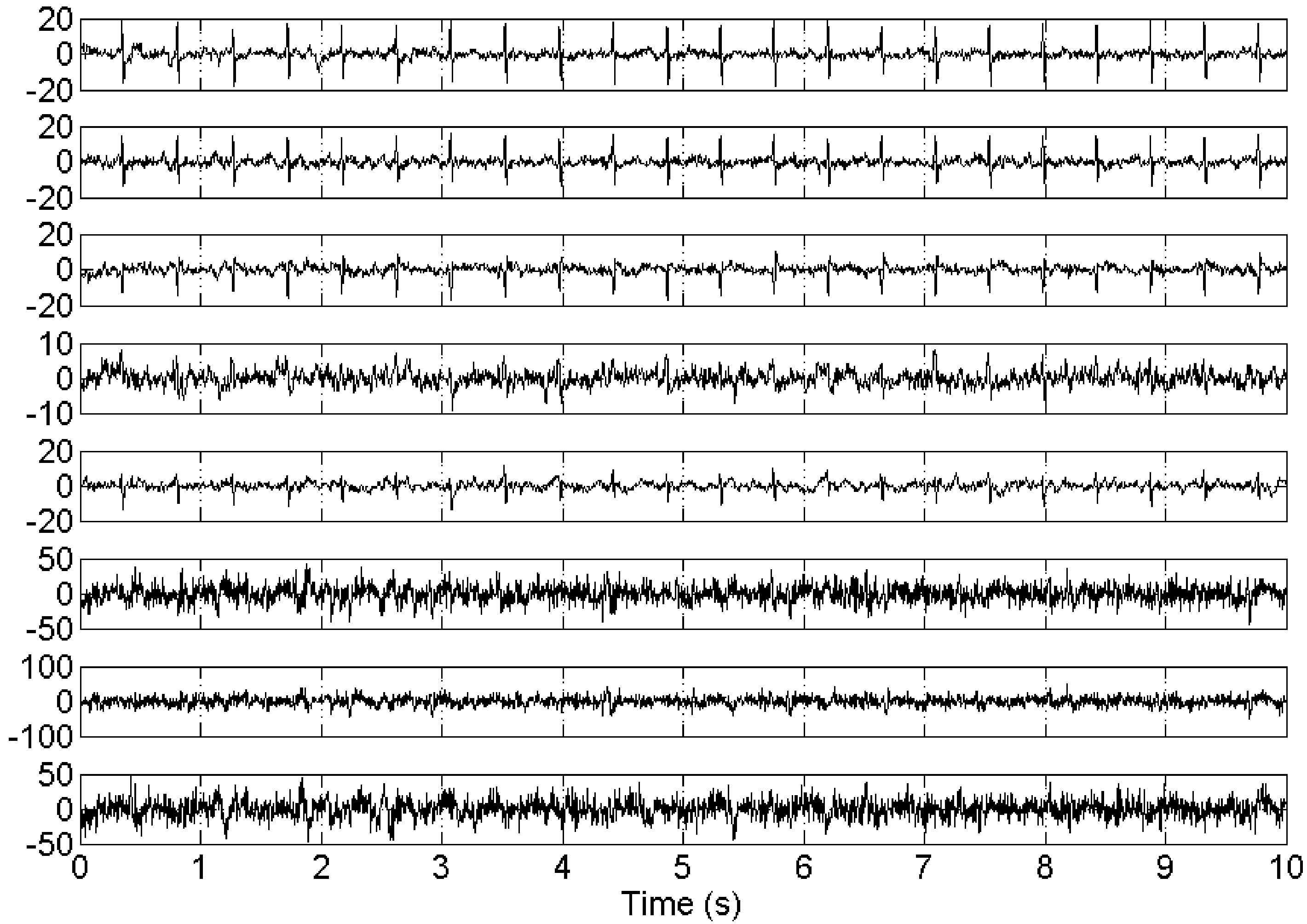
(continued)

Example 1: After 4th iteration



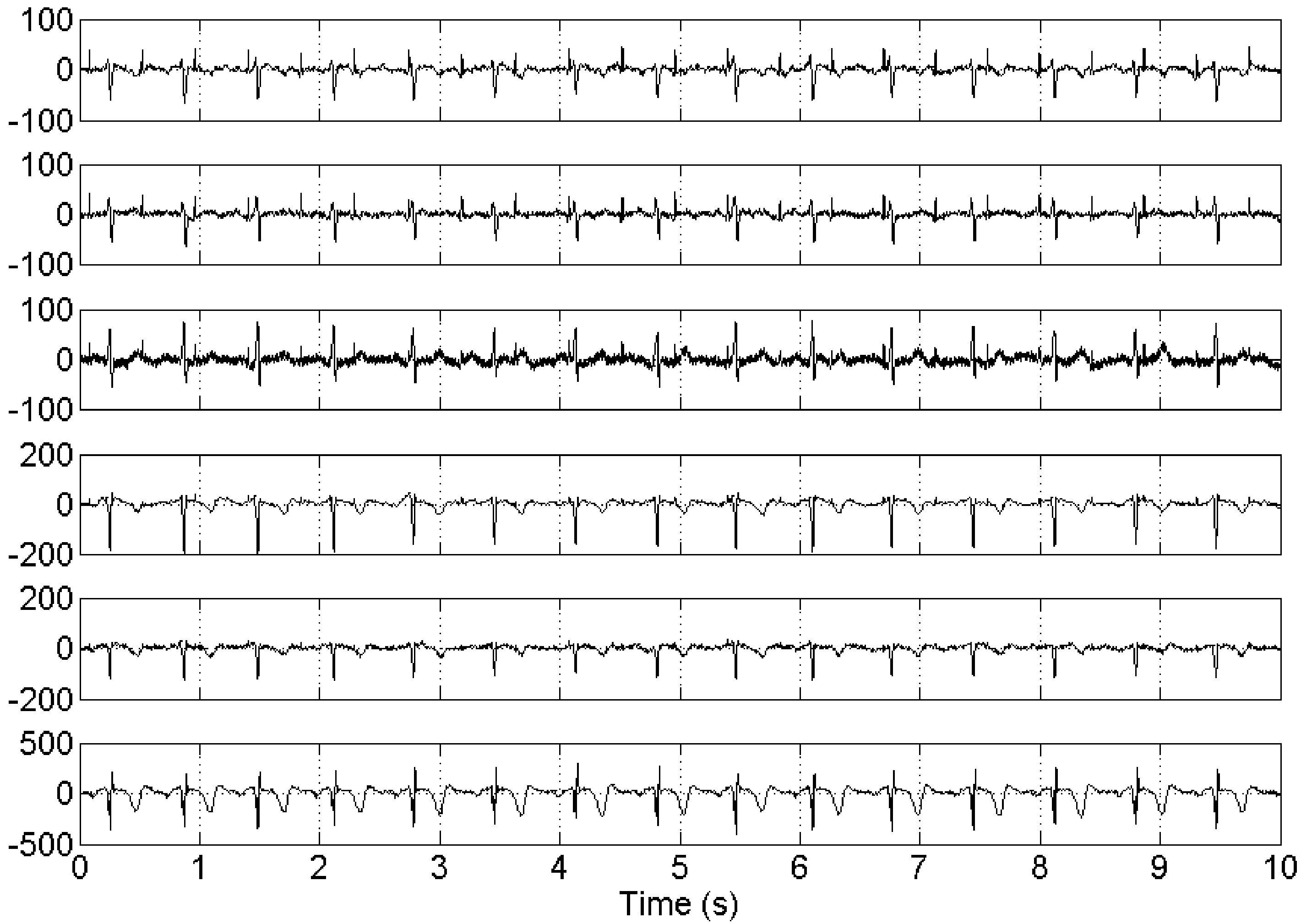
A Deflation Procedure for fECG Extraction
(continued)

Example 1: After 5th iteration (maternal ECG fully removed)



A Deflation Procedure for fECG Extraction
(continued)

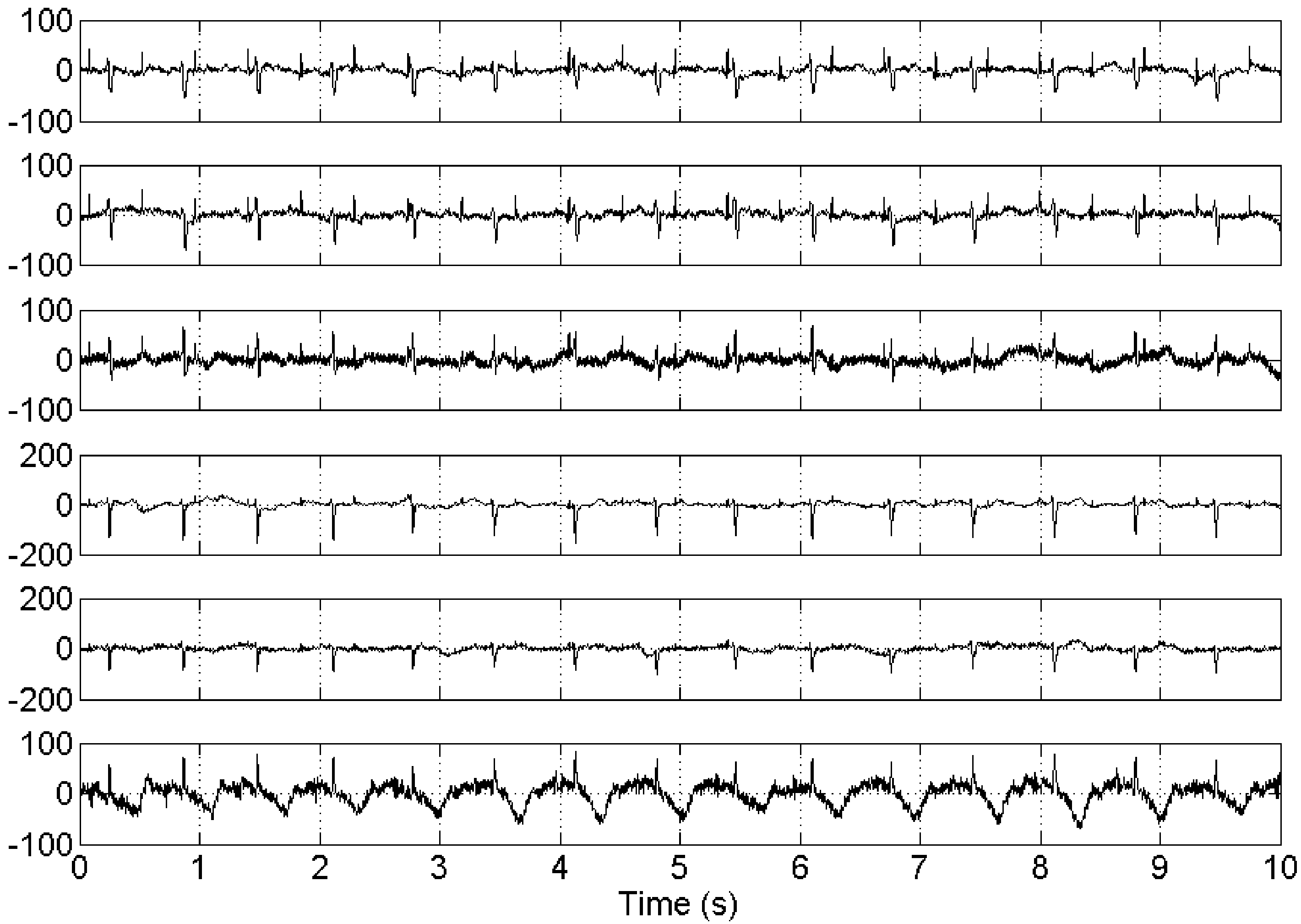
Example 2: A singular mixture (conventional ICA failed on this data)



A Deflation Procedure for fECG Extraction

(continued)

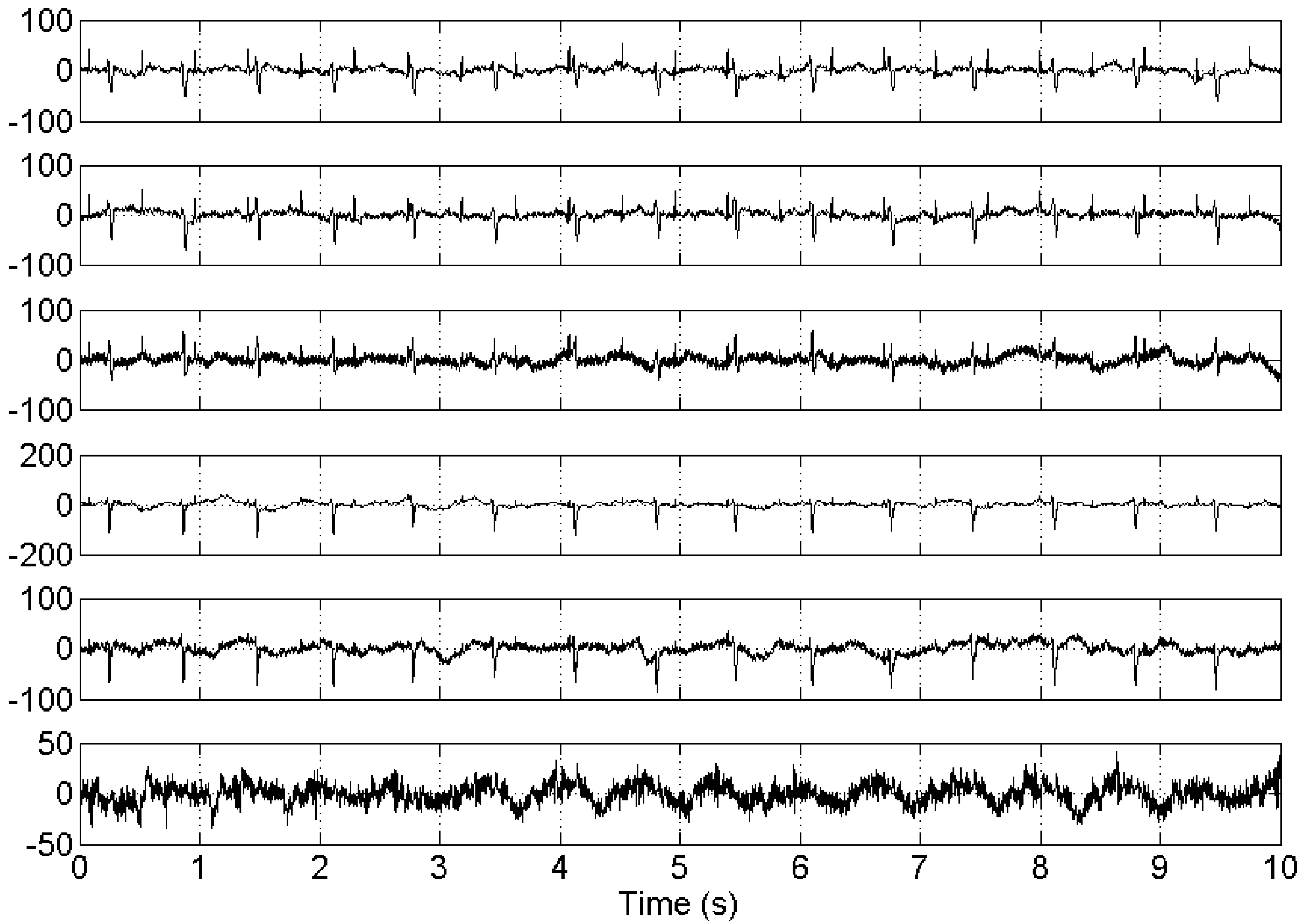
Example 2: After 1st iteration



A Deflation Procedure for fECG Extraction

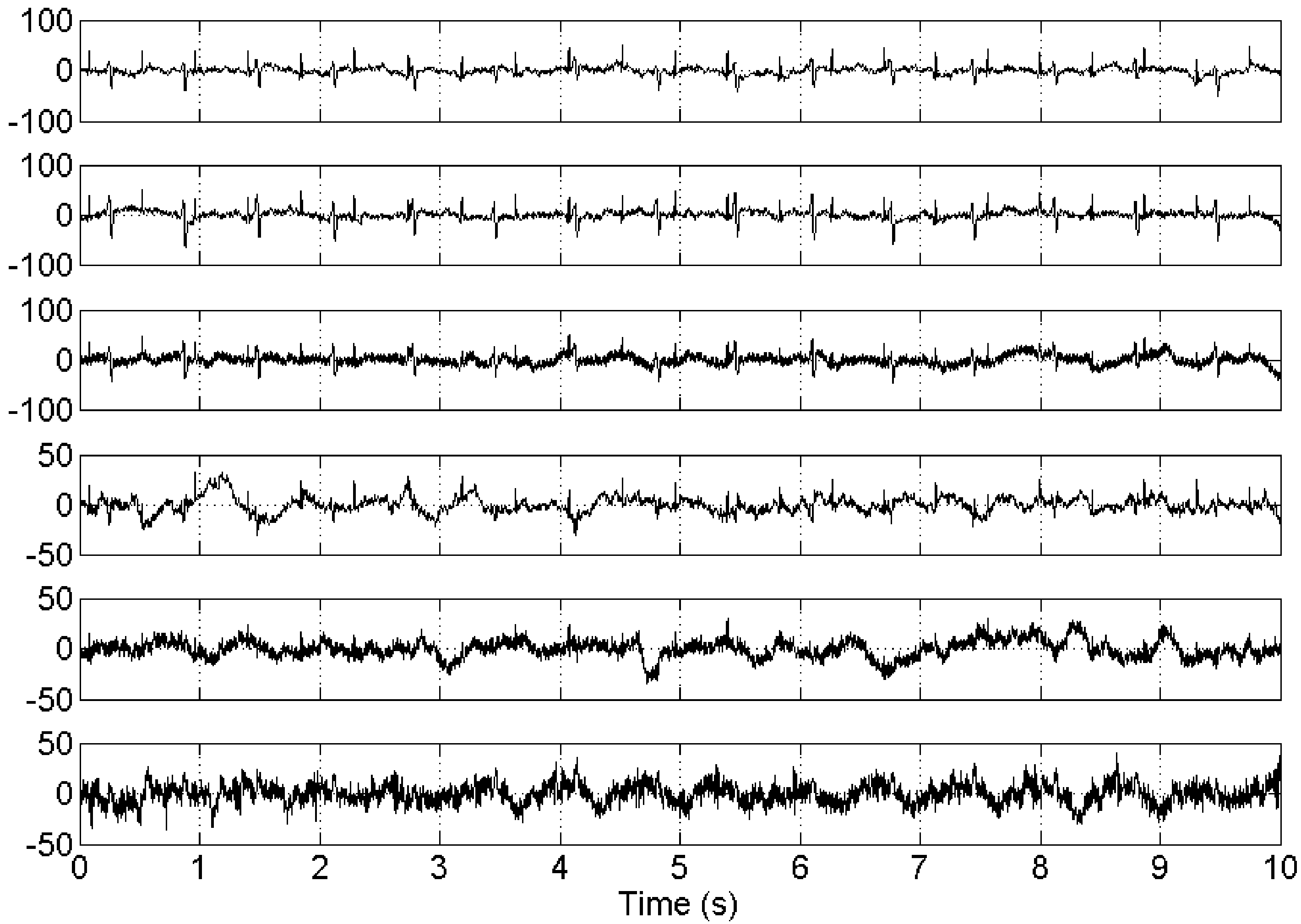
(continued)

Example 2: After 2nd iteration



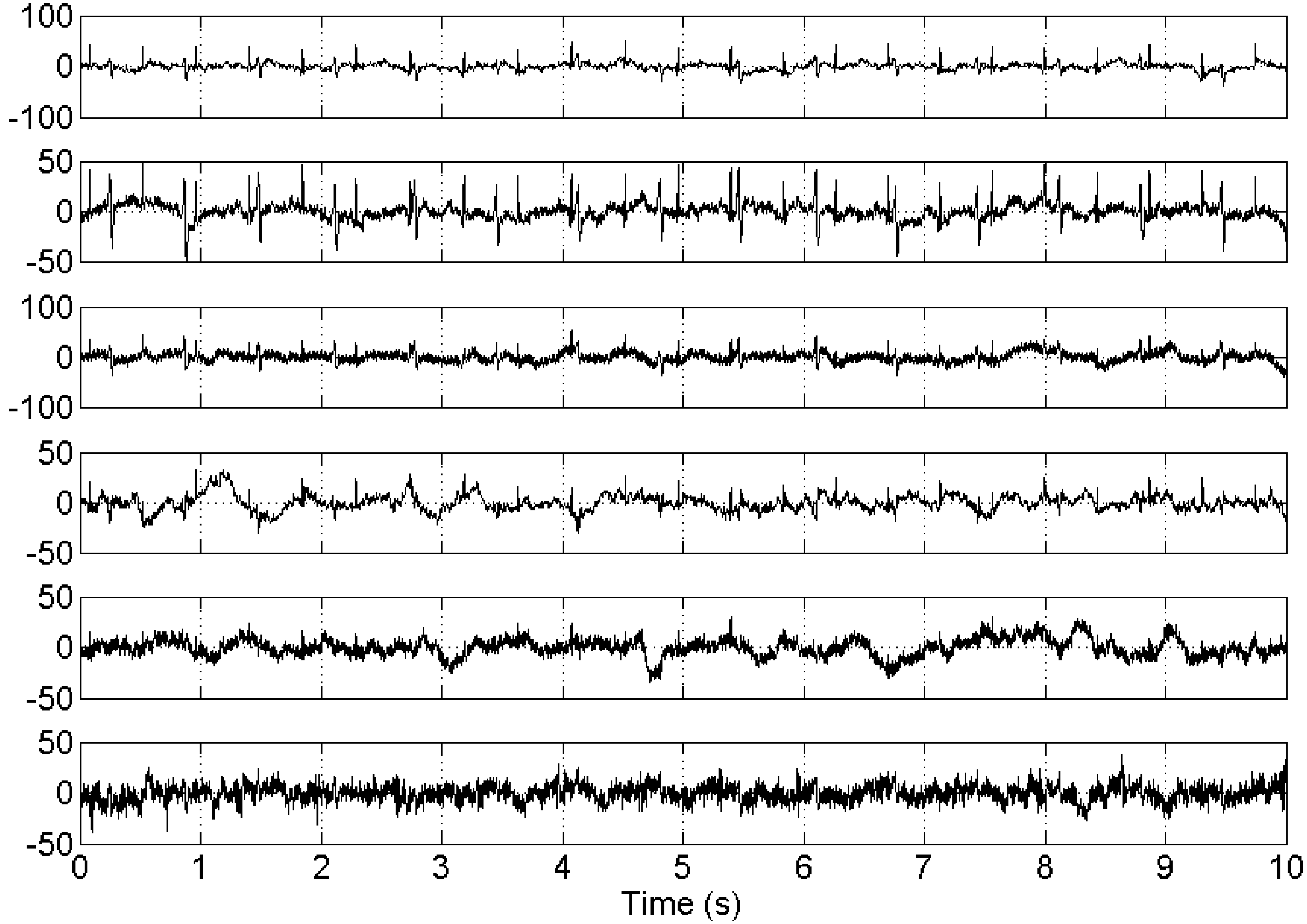
A Deflation Procedure for fECG Extraction
(continued)

Example 2: After 3rd iteration



A Deflation Procedure for fECG Extraction
(continued)

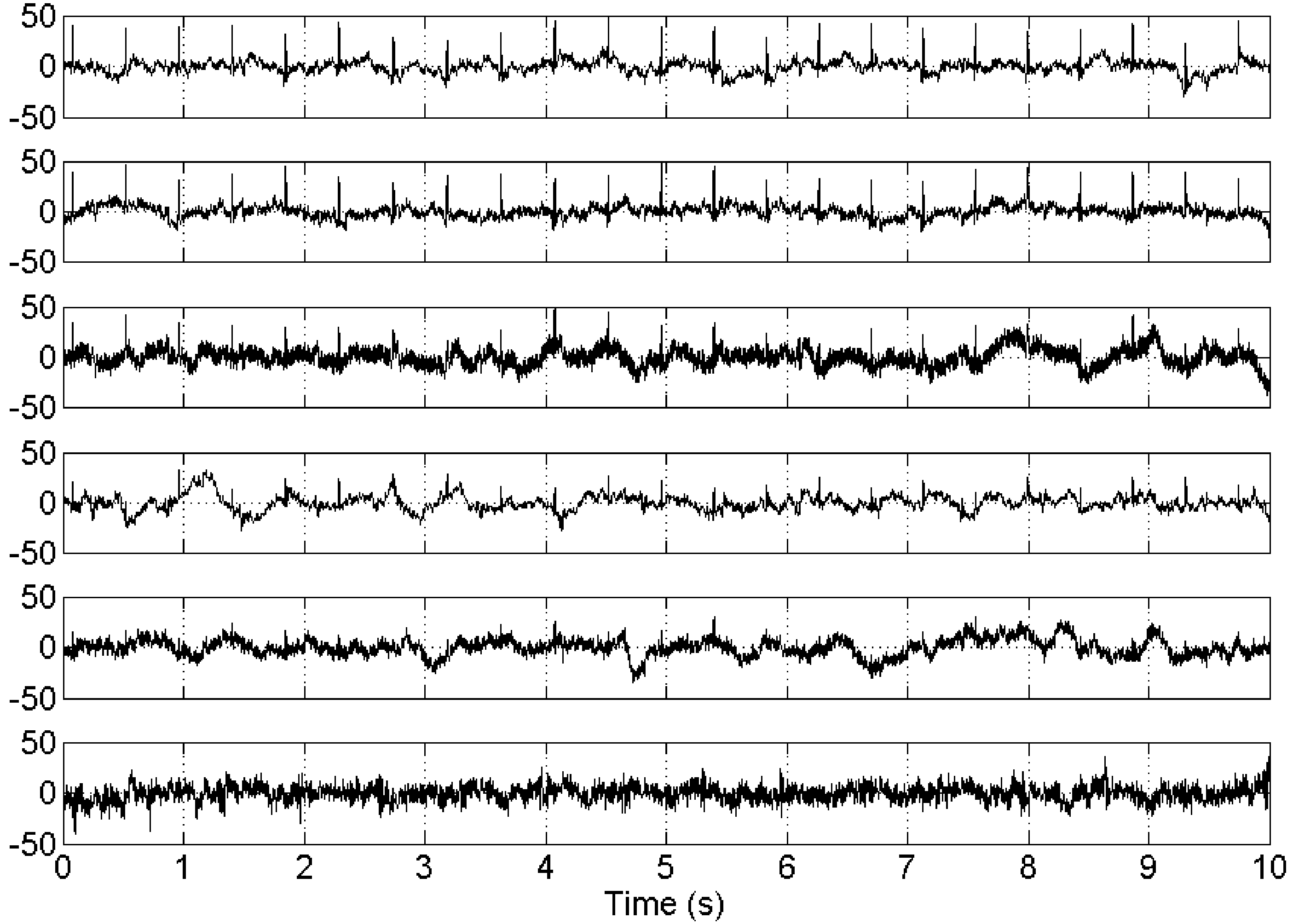
Example 2: After 4th iteration



A Deflation Procedure for fECG Extraction

(continued)

Example 2: After 5th iteration (maternal ECG fully removed)



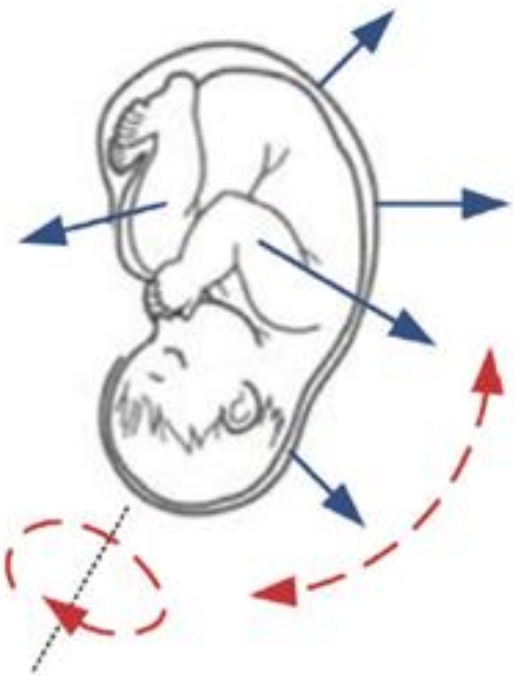
Time-Variant Scenarios

- Time-variant mixtures are another challenge for fECG separation
- Due to fetal movements, temporal variations are unavoidable in long recordings (lasting more than several minutes, depending on the fetal awareness)
- In this case, the noiseless data model is:

$$x(t)=H(t)s(t)$$

Therefore:

$$y(t)=Bx(t)=BH(t)s(t)\neq s(t).$$

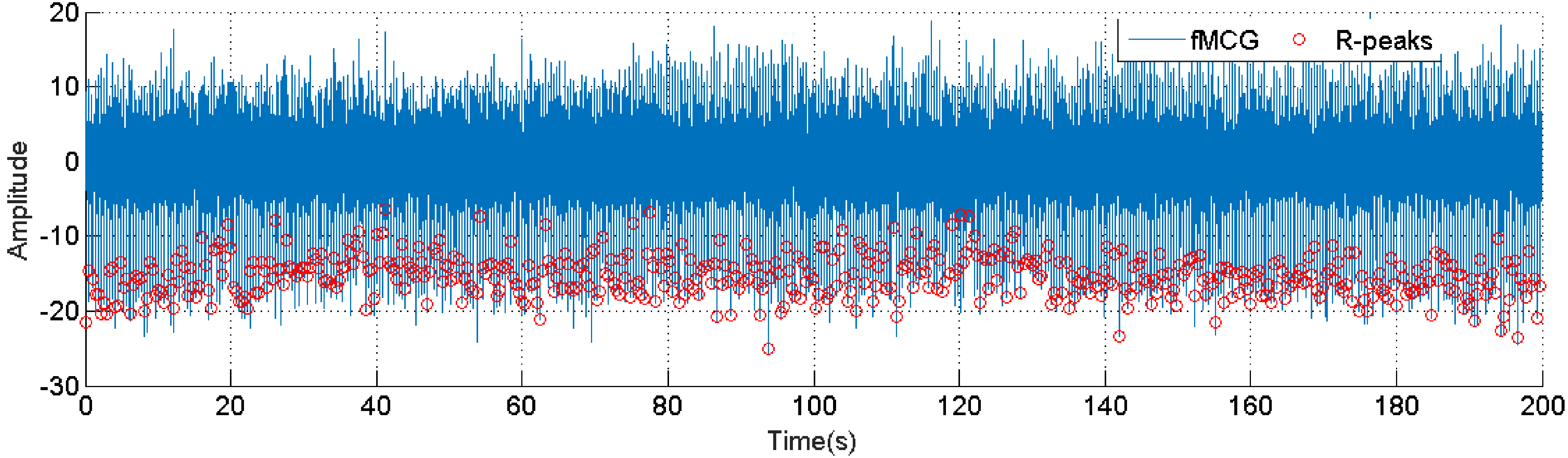


- Applying ICA, πCA, or other time-invariant algorithms to such data results in:

1. Partial separation of the sources
2. Time-varying sources which move from of channel to another

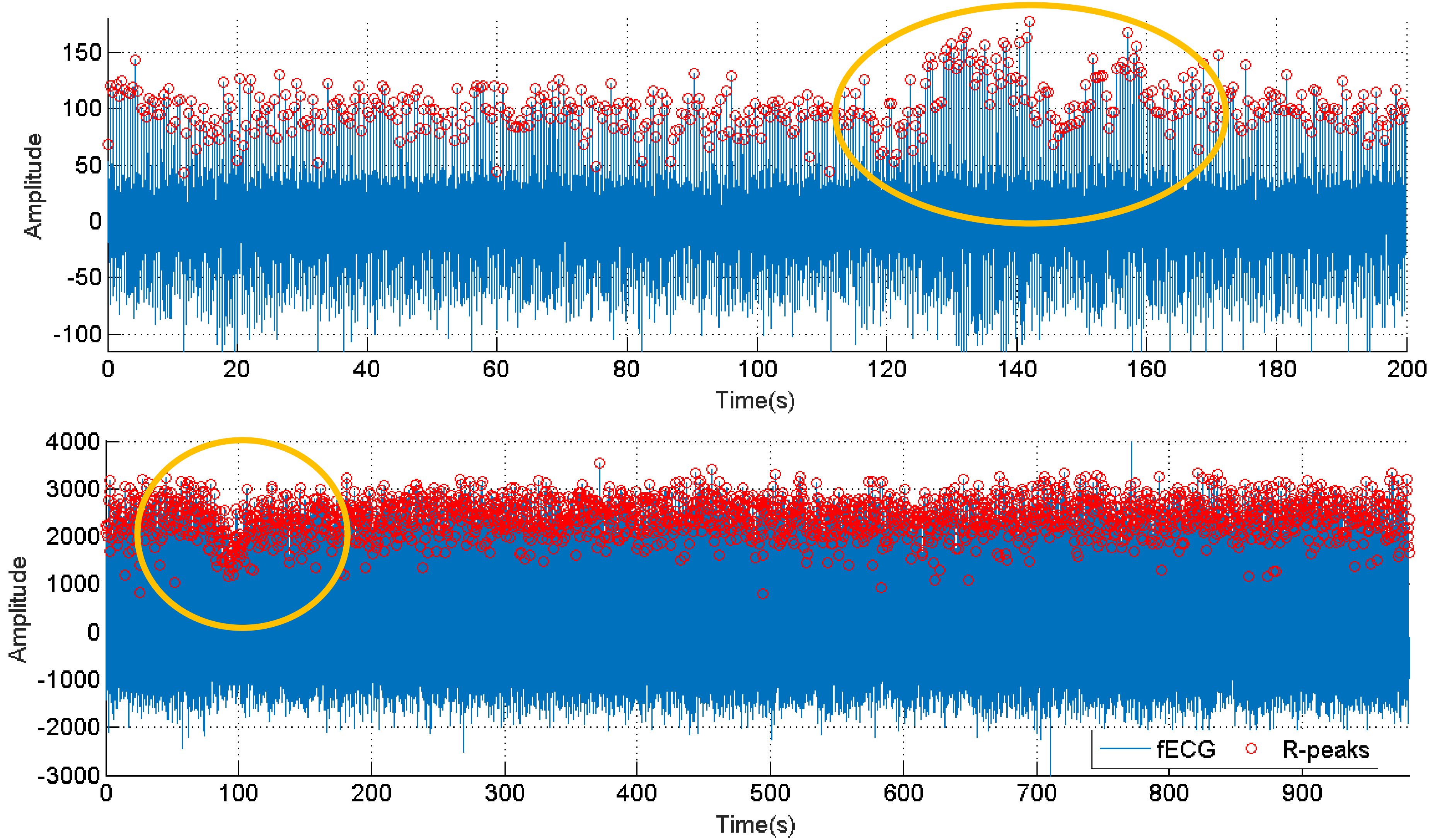
Time-Variant Scenarios

Example 1: Stationary case (for fetal MCG); no significant fetal motion observed



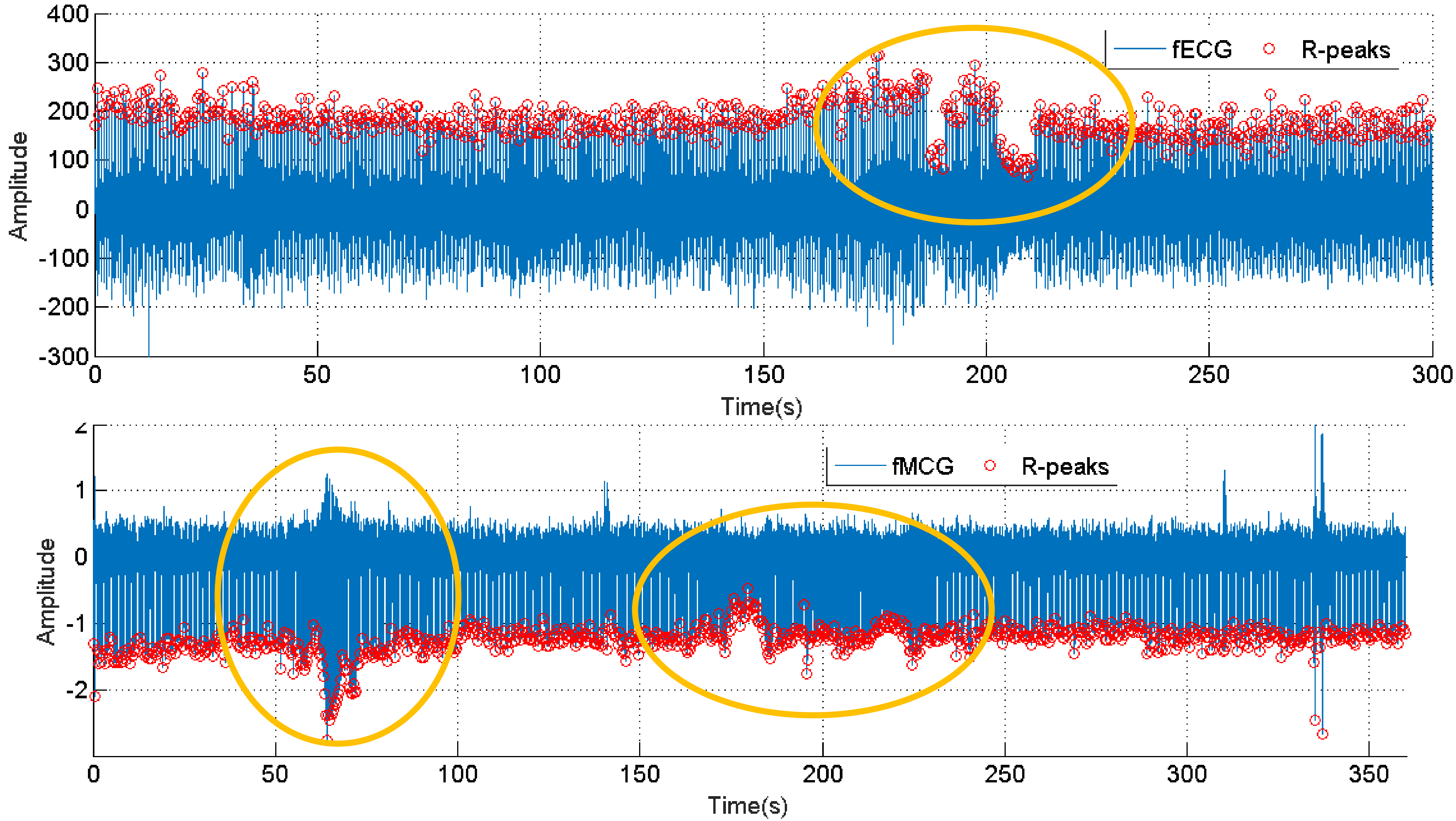
Time-Variant Scenarios

Example 2: Nonstationary case; minor fetal movement



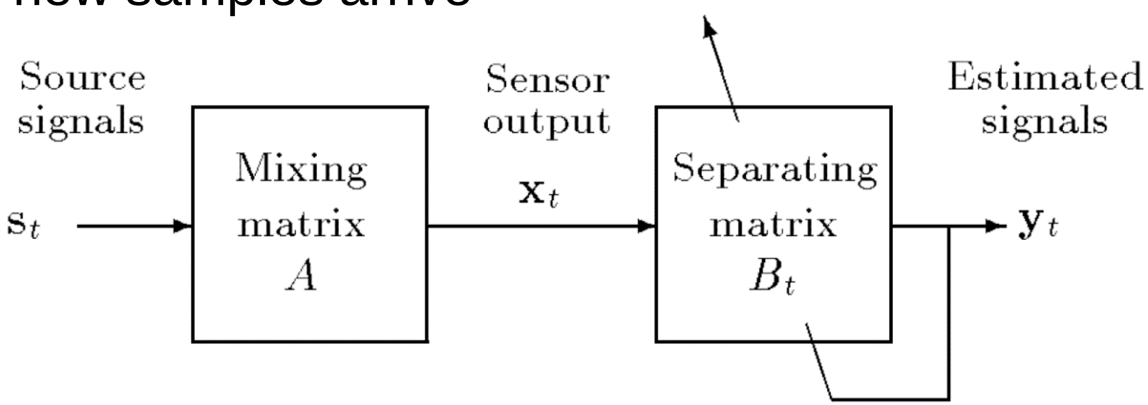
Time-Variant Scenarios

Example 3: Nonstationary case; significant fetal movement

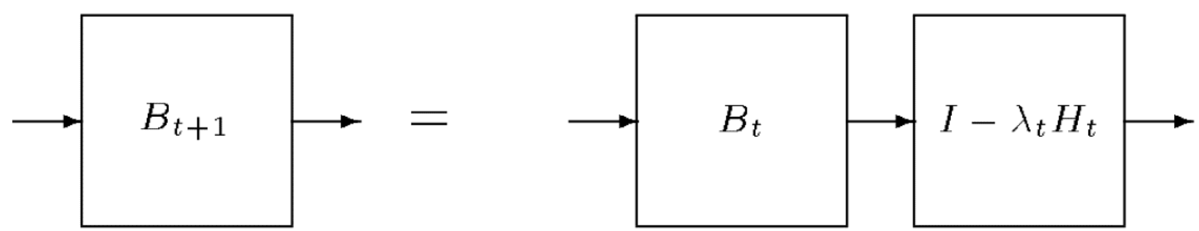


Online Source Separation

- In order to solve the issue of temporal variations in long maternal abdominal recordings (due to fetal motion), online tracking algorithms can be combined with source separation algorithms
- A well-known algorithm for online source separation is called equivariant adaptive separation via independence (EASI)
- This algorithm uses a serial (incremental) update of the demixing matrix as the new samples arrive



Ref: Cardoso, J. F., & Laheld, B. H. (1996). Equivariant adaptive source separation. *IEEE Transactions on signal processing*, *44*(12), 3017-3030.



FECG Tracking using Online Deflation

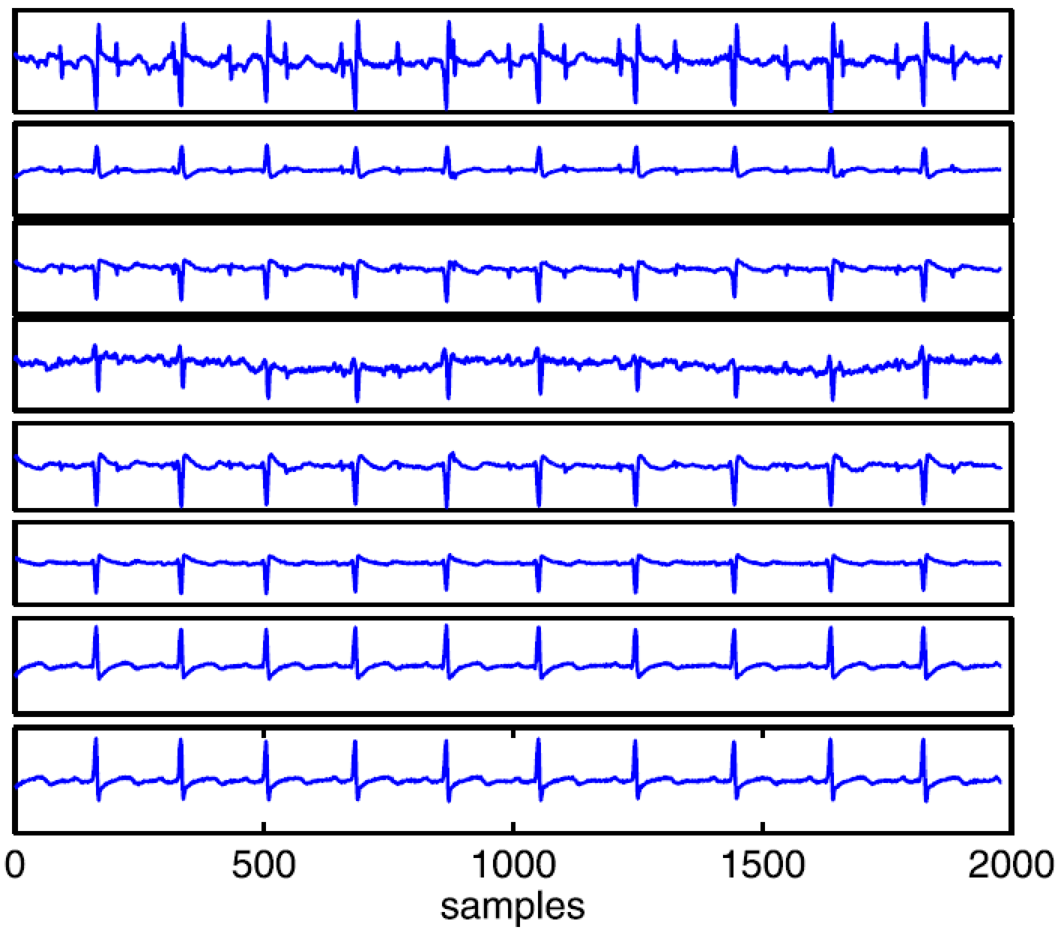
Algorithm 1 Online denoising by deflation (ODEFL)

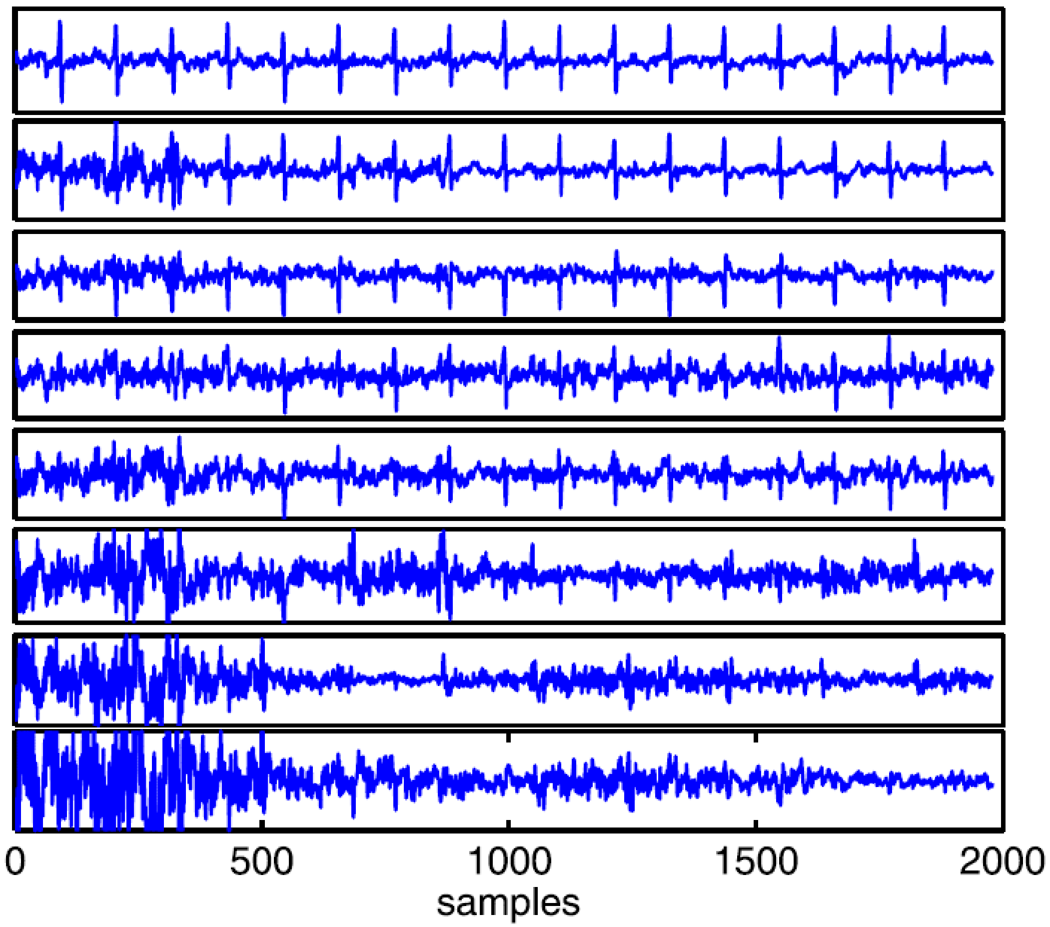
$\mathbf{x}_1(t) \leftarrow \mathbf{x}(t)$ ▷ Initialize with the input data
for $i = 1, \ldots, K$ **do** ▷ In each of the parallel stages of ODEFL
  $\mathbf{C}_i(0) \leftarrow \mathbf{I}_N$ ▷ Initialize with identity (or random unitary) matrices
  $\mathbf{C}_{\tau,i}(0) \leftarrow \mathbf{I}_N$
  $\mathbf{W}_i(0) = [\mathbf{w}_{i1}(0), \ldots, \mathbf{w}_{iN}(0)] \leftarrow \mathbf{I}_N$
  for $t = 1, \ldots, T$ **do** ▷ Repeat for all samples of the data
    $\mathbf{C}_i(t) \leftarrow \beta \mathbf{C}_i(t-1) + \mathbf{x}_i(t)\mathbf{x}_i(t)^T$ ▷ Covariance matrix update
    $\mathbf{C}_{\tau,i}(t) \leftarrow \gamma \mathbf{C}_{\tau,i}(t-1) + \mathbf{x}_i(t)\mathbf{x}_i(t+\tau_t)^T$ ▷ Lagged covariance matrix update
    $\mathbf{C}_i(t) \leftarrow [\mathbf{C}_i(t) + \mathbf{C}_i(t)^T]/2$ ▷ Covariance matrix symmetrization
    $\mathbf{C}_{\tau,i}(t) \leftarrow [\mathbf{C}_{\tau,i}(t) + \mathbf{C}_{\tau,i}(t)^T]/2$ ▷ Lagged covariance matrix symmetrization
    $\mathbf{C}_{\tau,i}^{-1}(t) \leftarrow \gamma^{-1}\mathbf{C}_{\tau,i}^{-1}(t-1) - \dfrac{\gamma^{-1}\mathbf{C}_{\tau,i}^{-1}(t-1)\mathbf{x}_i(t)\mathbf{x}_i(t+\tau_t)^T\mathbf{C}_{\tau,i}^{-1}(t-1)}{\gamma + \mathbf{x}_i(t)^T\mathbf{C}_{\tau,i}^{-1}(t-1)\mathbf{x}_i(t+\tau_t)}$ ▷ Matrix inversion lemma
    $\tilde{\mathbf{C}} \leftarrow \mathbf{C}_i(t)$
    for $j = 1, \ldots, N$ **do** ▷ Perform Online GEVD of $(\tilde{\mathbf{C}}, \mathbf{C}_\tau(t))$ over all channels
      $\mathbf{w}_{ij}(t) \leftarrow \dfrac{\mathbf{w}_{ij}^T(t-1)\mathbf{C}_{\tau,i}(t)\mathbf{w}_{ij}(t-1)}{\mathbf{w}_{ij}^T(t-1)\tilde{\mathbf{C}}\mathbf{w}_{ij}(t-1)}\mathbf{C}_{\tau,i}^{-1}(t)\tilde{\mathbf{C}}\mathbf{w}_{ij}(t-1)$ ▷ Incremental CSP update
      $\mathbf{w}_{ij}(t) \leftarrow \mathbf{w}_{ij}(t)/\|\mathbf{w}_{ij}(t)\|$ ▷ Normalization
      $\tilde{\mathbf{C}} \leftarrow \left[\mathbf{I}_N - \dfrac{\tilde{\mathbf{C}}\mathbf{w}_{ij}\mathbf{w}_{ij}^T}{\mathbf{w}_{ij}^T\tilde{\mathbf{C}}\mathbf{w}_{ij}}\right]\tilde{\mathbf{C}}$ ▷ Deflation procedure
    end for
    $\mathbf{W}_i(t) = [\mathbf{w}_{i1}(t), \ldots, \mathbf{w}_{iN}(t)]$
    $\mathbf{s}(t) \leftarrow \mathbf{W}_i^T(t)\mathbf{x}_i(t)$ ▷ Go to transform space
    Estimate L ▷ The number of effective mECG dimensions
    $\tilde{\mathbf{s}}(t) \leftarrow \mathbf{G}_i(\mathbf{s}(t), L)$ ▷ Apply denoising operator over the first L channels
    $\mathbf{y}_i(t) \leftarrow \mathbf{W}_i^{-T}(t)\tilde{\mathbf{s}}(t)$ ▷ Return to original space
    $\mathbf{x}_{i+1}(t) \leftarrow \mathbf{y}_i(t)$ ▷ Use output as input for next stage
  end for
end for

Ref: M. Fatemi and R. Sameni, “An Online Subspace Denoising Algorithm for Maternal ECG Removal from Fetal ECG Signals,” *Iranian Journal of Science and Technology, Transactions of Electrical Engineering*, vol. 2017, pp. 1-15, April 2017.

FECG Tracking using Online Deflation

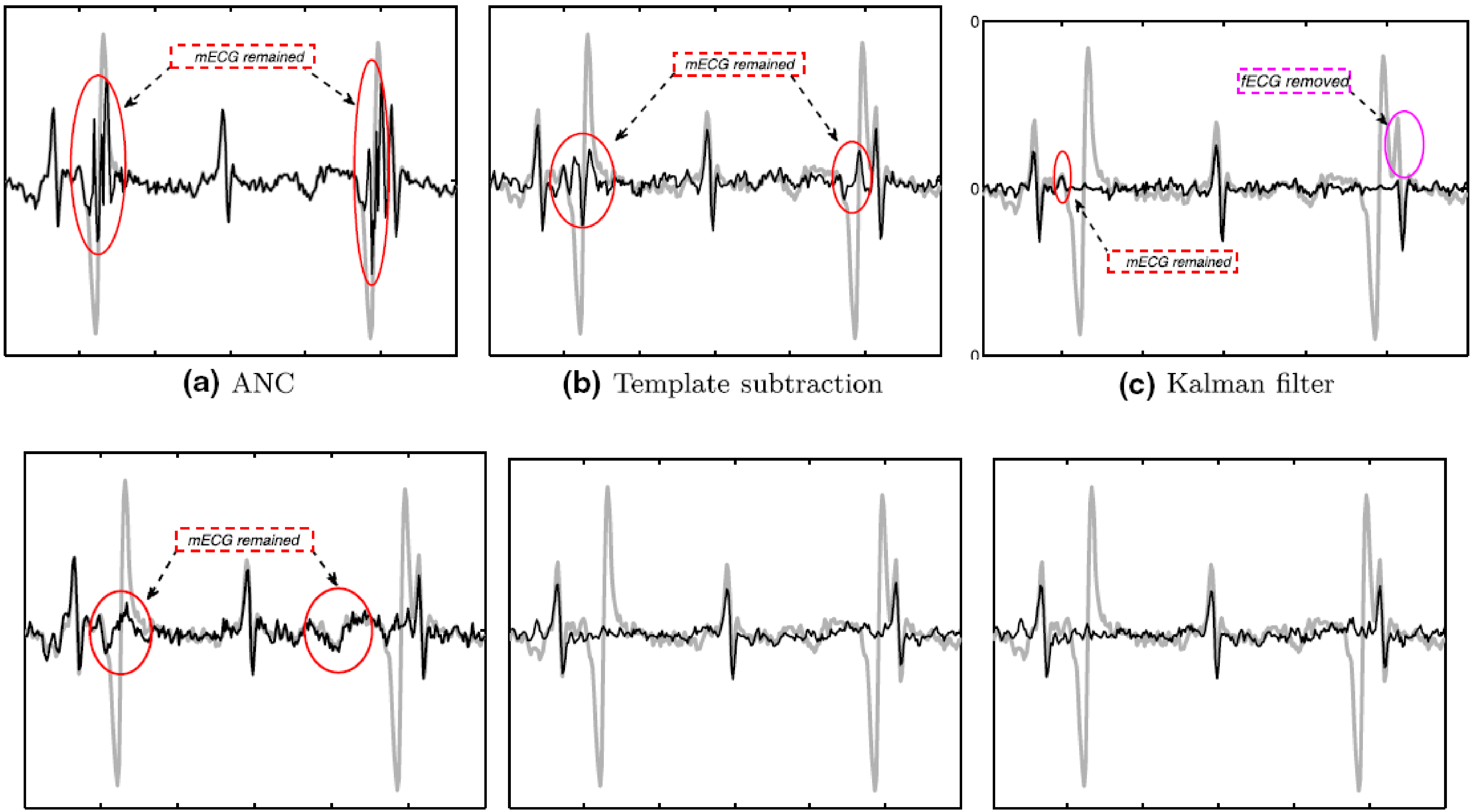
Example:





Comparison of Different Methods

Example:



(a) ANC **(b)** Template subtraction **(c)** Kalman filter

(d) BSS **(e)** DEFL **(f)** ODEFL

PART I

Post-Processing, Parameter Extraction and Advanced Topics

Post-Processing

The Steps After Fetal ECG Extraction

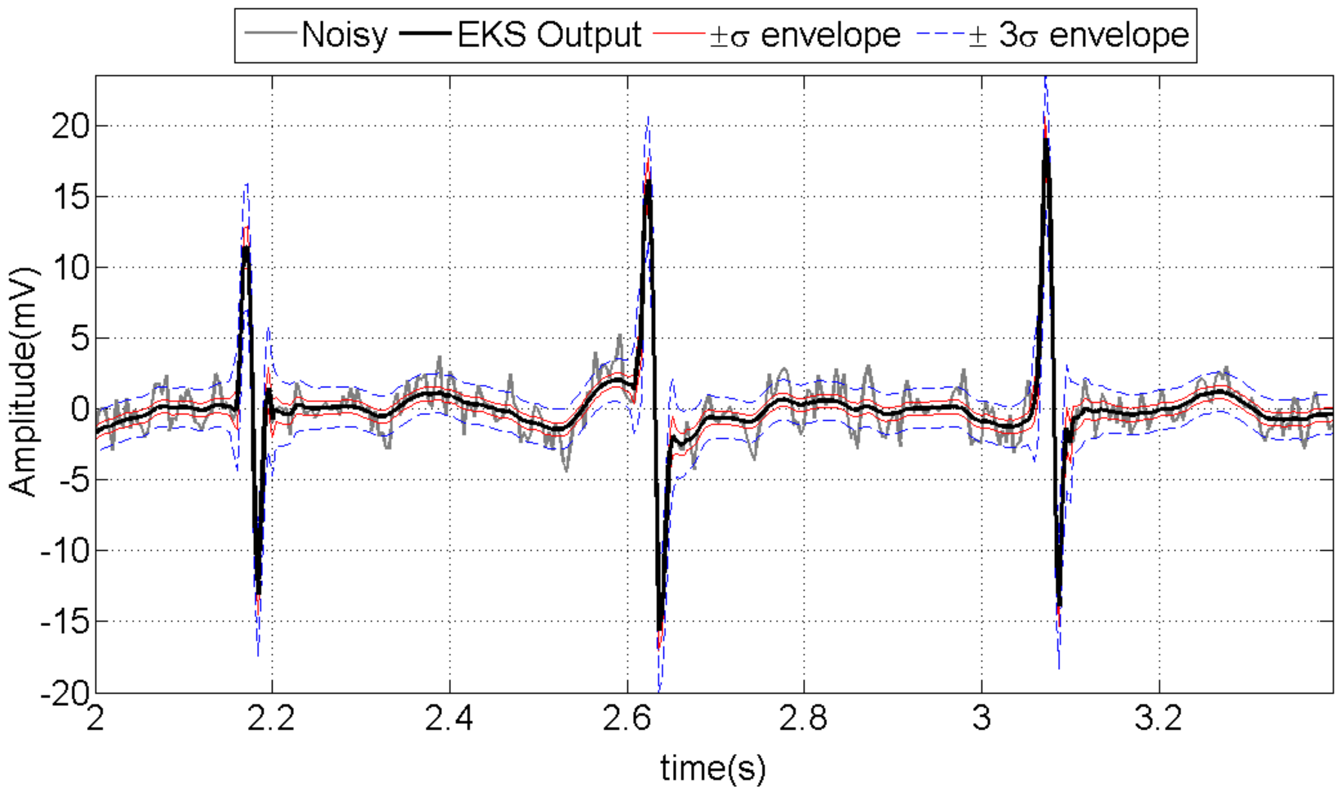
Post-Extraction Filtering

- The fECG resulting from mECG cancellation is still very noisy and requires post-processing
- The extracted fECG may be processed in the original domain (sensor domain) or in the transform domain (e.g., space of independent components)
- Different post-processings include:
  - Simple bandpass filtering
  - Wavelet denoising
  - Adaptive noise cancellation
  - Kalman filtering
  - …

Identical to the methods used for adults; but with customized set of parameters

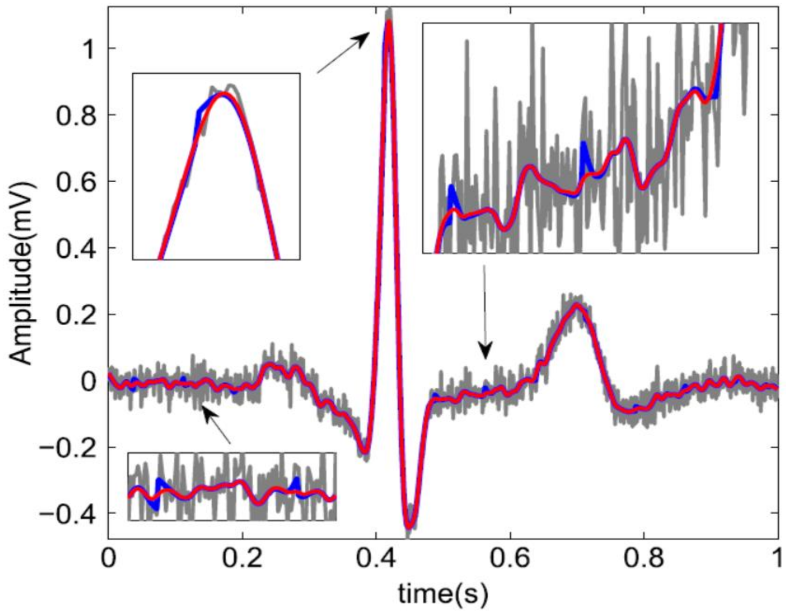
Post FECG Extraction Kalman Smoothing

- The extended Kalman filtering framework detailed before can be used for post-processing the extracted fetal ECG.
- An additional benefit of the Kalman filter is the estimation accuracy margins obtained from the estimates' covariance matrices.



FECG Denoising using Smoothness Priors

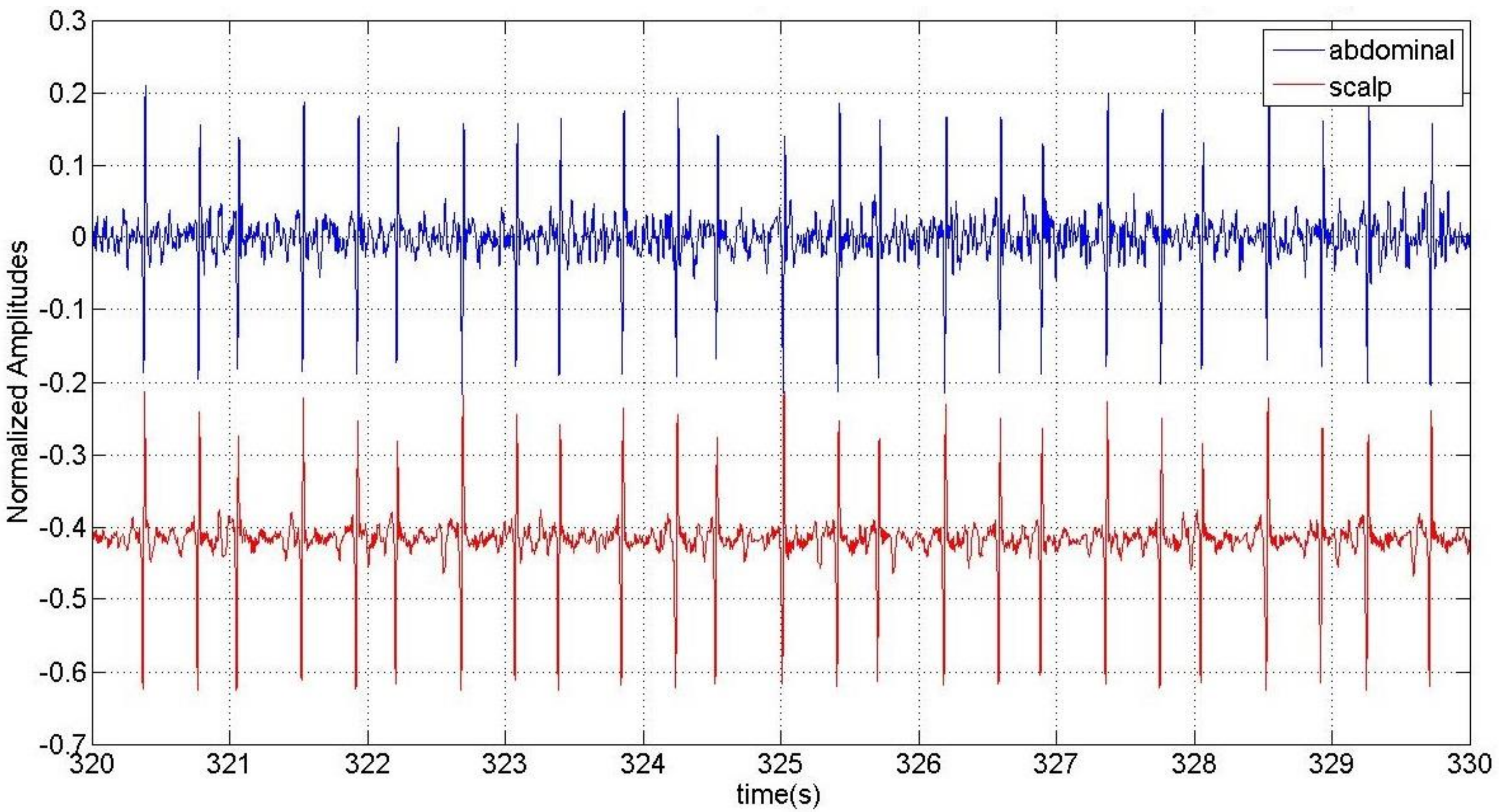
- An alternative method for fECG post-processing, which does not require prior dynamics is based on piece-wise smoothness priors.
- This approach has been shown to be closely related to wavelet denoising, Wiener filter and Tikhonov regularization.



- Ref: Sameni, R. (2017). Online filtering using piecewise smoothness priors: Application to normal and abnormal electrocardiogram denoising. *Signal Processing*, *133*, 52-63.
- An old wavelet-based method for fECG extraction: A. Khamene and S. Negahdaripour, "A new method for the extraction of fetal ECG from the composite abdominal signal," *Biomedical Engineering, IEEE Transactions on*, vol. 47, no. 4, pp. 507-516, April 2000.

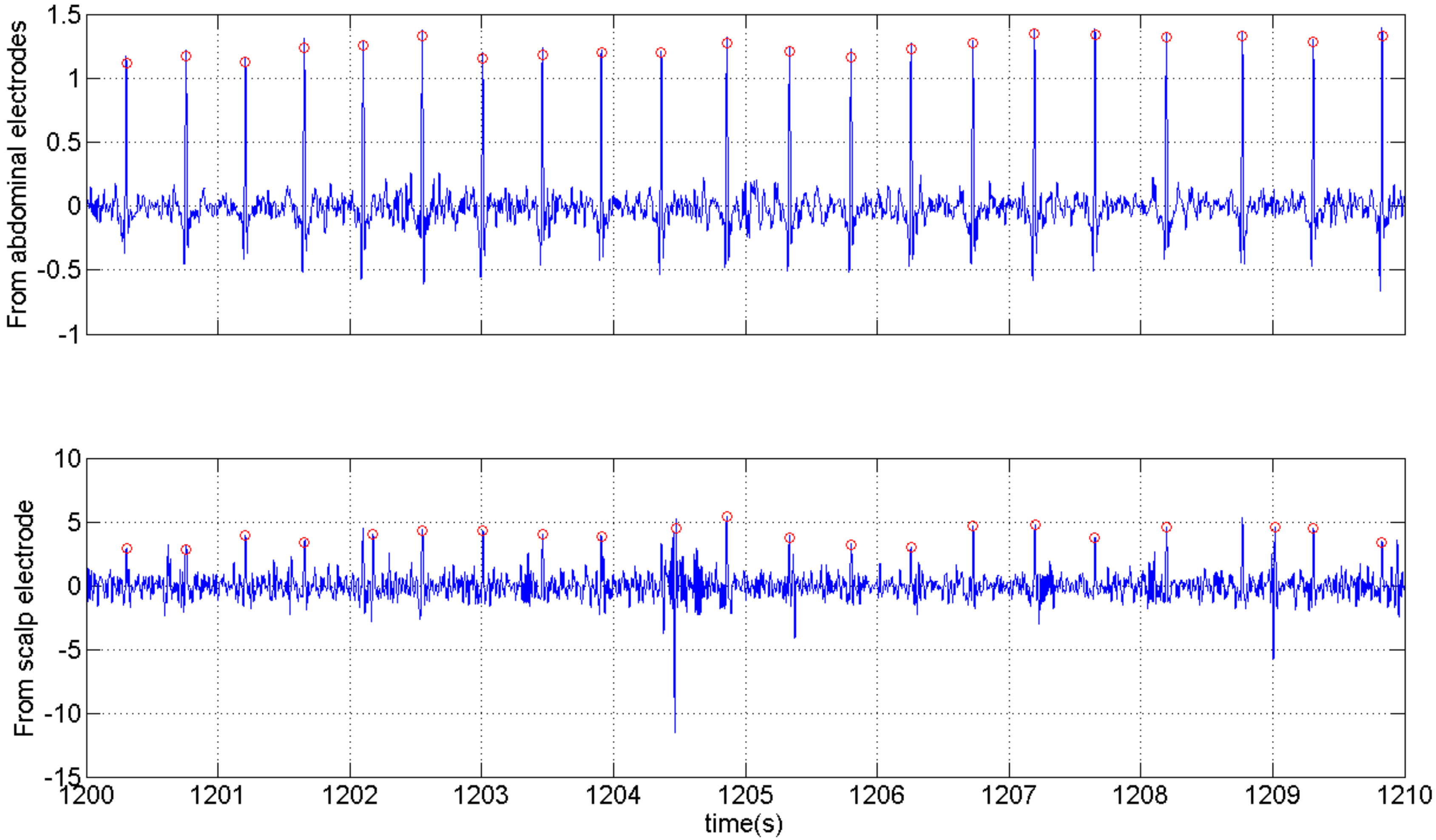
Maternal Abdominal vs. Fetal Scalp Lead Results

Example 1:



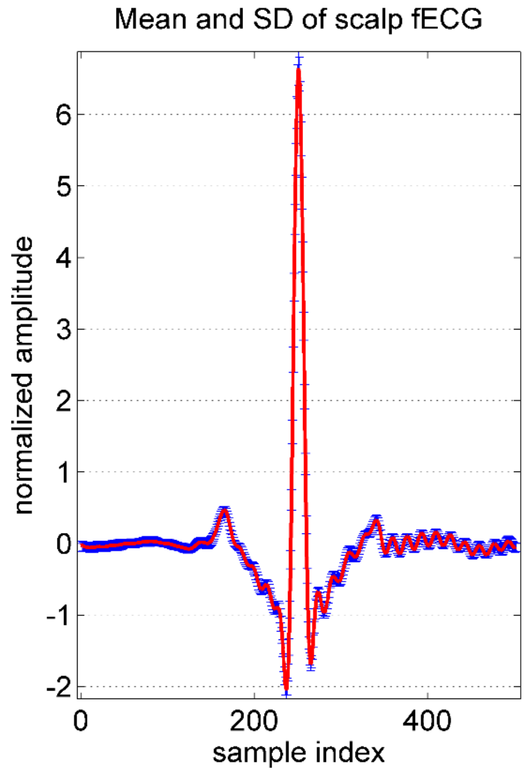
Maternal Abdominal vs. Fetal Scalp Lead Results

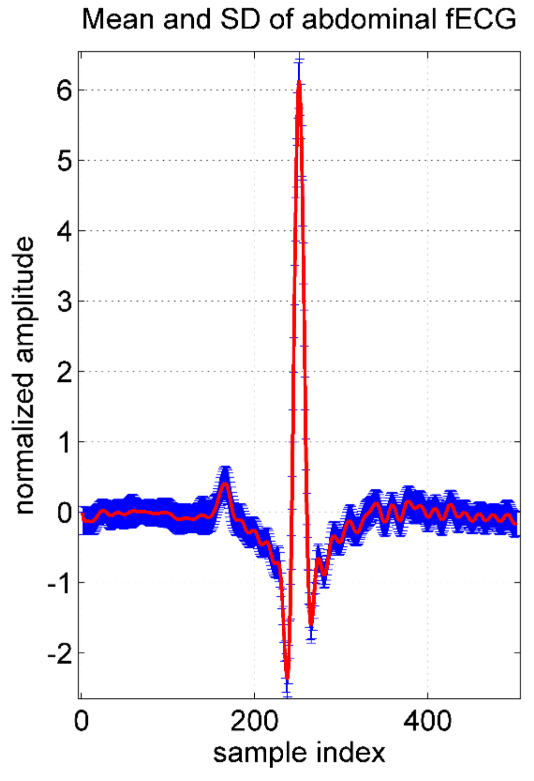
Example 2: The abdominal results may even be better than the scalp lead results in some cases

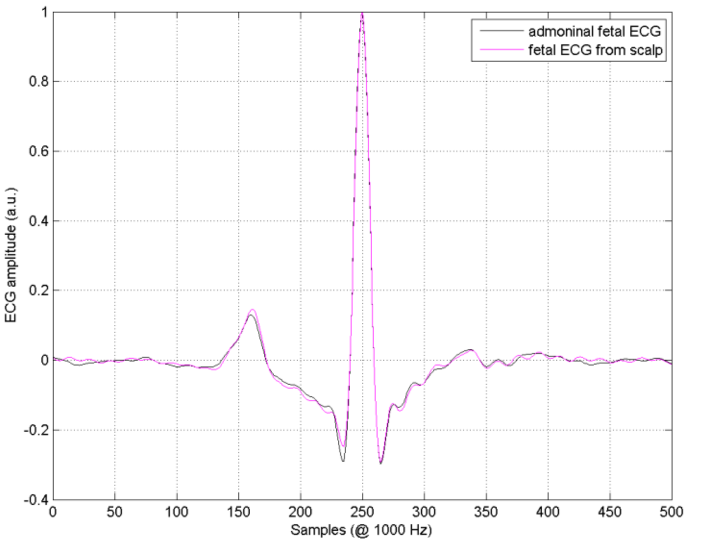


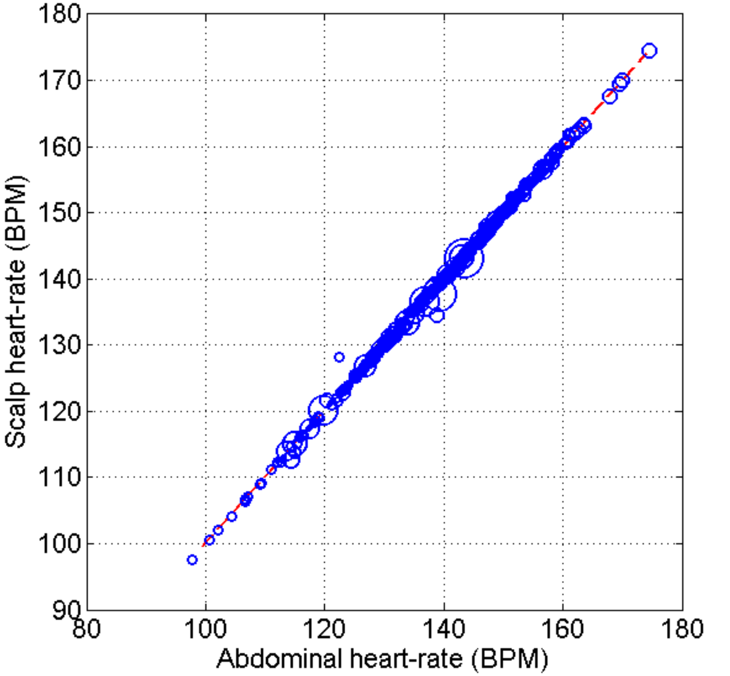
Maternal Abdominal vs. Fetal Scalp Lead Results (continued)

Example:









Ref: Clifford, G., Sameni, R., Ward, J., Robinson, J., & Wolfberg, A. J. (2011). Clinically accurate fetal ECG parameters acquired from maternal abdominal sensors. *American Journal of Obstetrics & Gynecology*, *205* (1), 47-e1.

Fetal ECG Parameter Extraction

Post Fetal ECG Extraction Analysis

Fetal ECG Parameters

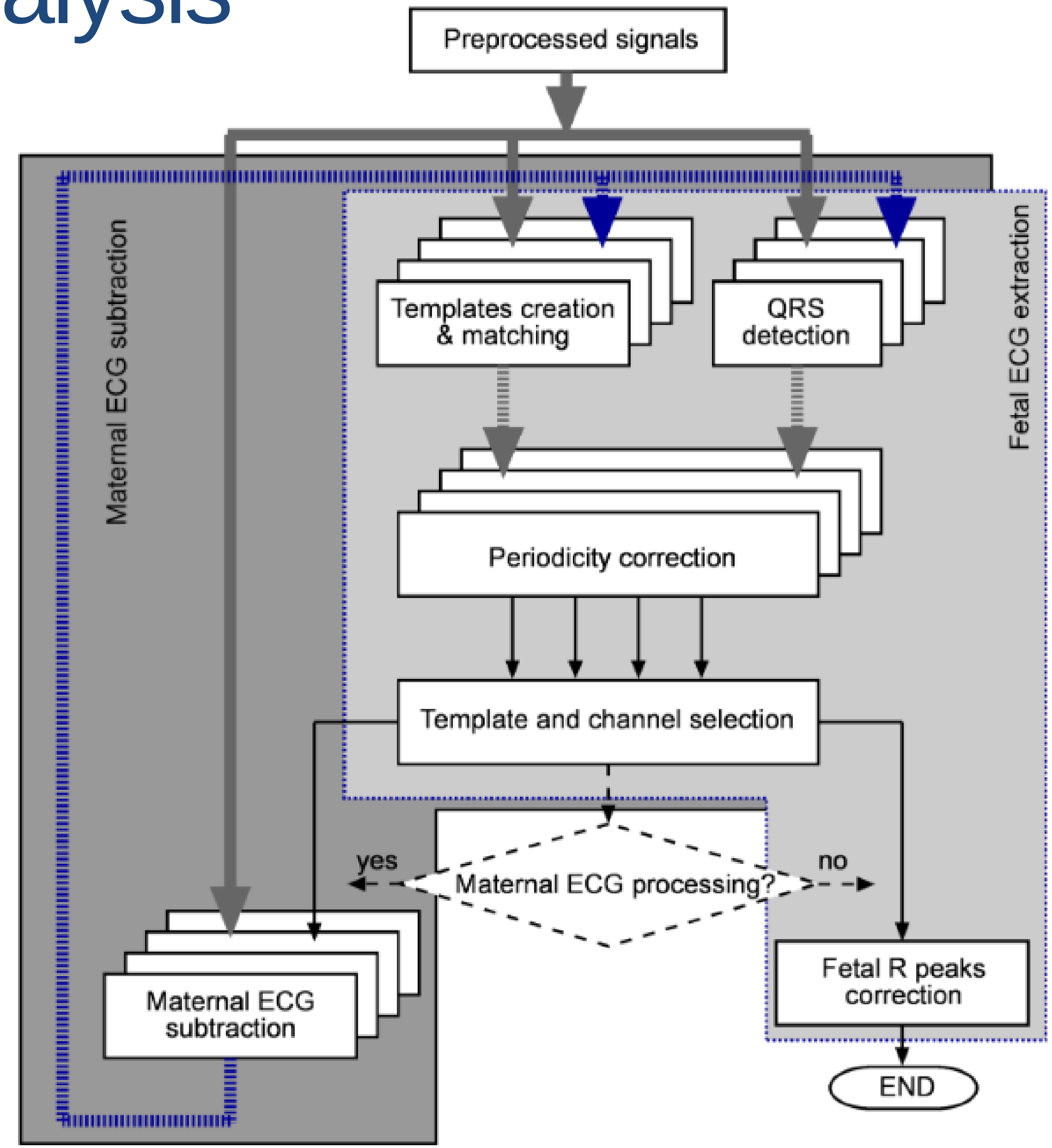
As with adult ECG, clinical information regarding the fetal heart is embedded in the heart rate and morphology of the fetal ECG:

1. Heart-rate: The fetal heart-rate time series is obtained by R-peak detection (as with adults). However, since the data is more noisy, robust algorithms such as matched filters with post-refining (eliminating excess or re-finding missing beats) can be used
2. Morphology: Fetal ECG morphologic parameters are commonly extracted by synchronous averaging to improve the SNR

Further reading: The Noninvasive Fetal ECG: the PhysioNet/Computing in Cardiology Challenge 2013: Silva, I., Behar, J., Sameni, R., Zhu, T., Oster, J., Clifford, G. D., & Moody, G. B. (2013, September). Noninvasive fetal ECG: the PhysioNet/computing in cardiology challenge 2013. In *Computing in Cardiology Conference (CinC), 2013* (pp. 149-152). IEEE.

Fetal Heart Rate Analysis

The fetal heart-rate extracted from the fECG commonly requires some rule-based corrections for missing beats or erroneous excess beat counts



Ref: Dessi, A., Pani, D., & Raffo, L. (2013, September). Identification of fetal QRS complexes in low density non-invasive biopotential recordings. In *Computing in Cardiology Conference (CinC),* (pp. 321-324). IEEE.

Fetal Heart Rate Correction

- Fetal heart-rate corrections can be made using matched filtering and conditional median filter as post processing
- Conditional median filtering for refining the heart-rate sequence:

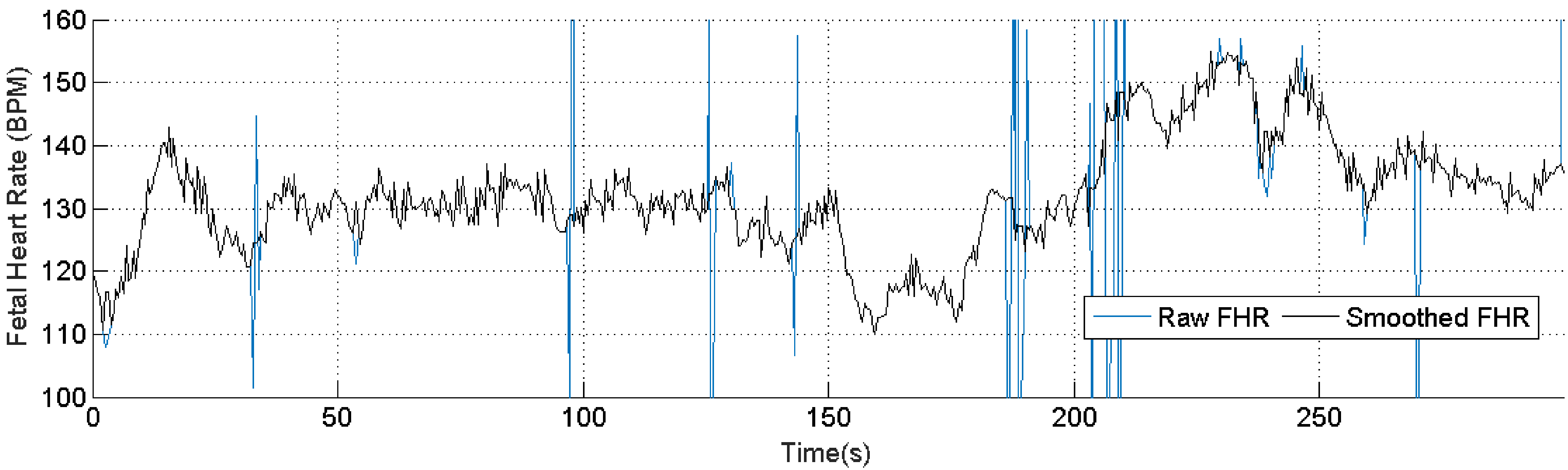
```
for n = 1,2,…,N
   med[n] = MEDIAN(HR[n-w], HR[n-w+1], …, HR[n+w])
   if(|HR[n] - med[n]| > Threshold)
       HRc[n] = med[n];
   else
       HRc[n] = HR[n];
 end
```

- This filter is nonlinear; it retains the HR sequence unchanged during normal segments and only smoothens the outliers.

Ref: Various other nonlinear filtering techniques can be used in this context. For further details, refer to: Arce, G. R. (2005). *Nonlinear signal processing: a statistical approach*. John Wiley & Sons.

Fetal Heart Rate Correction (continued)

Example 1: Fetal ECG refining



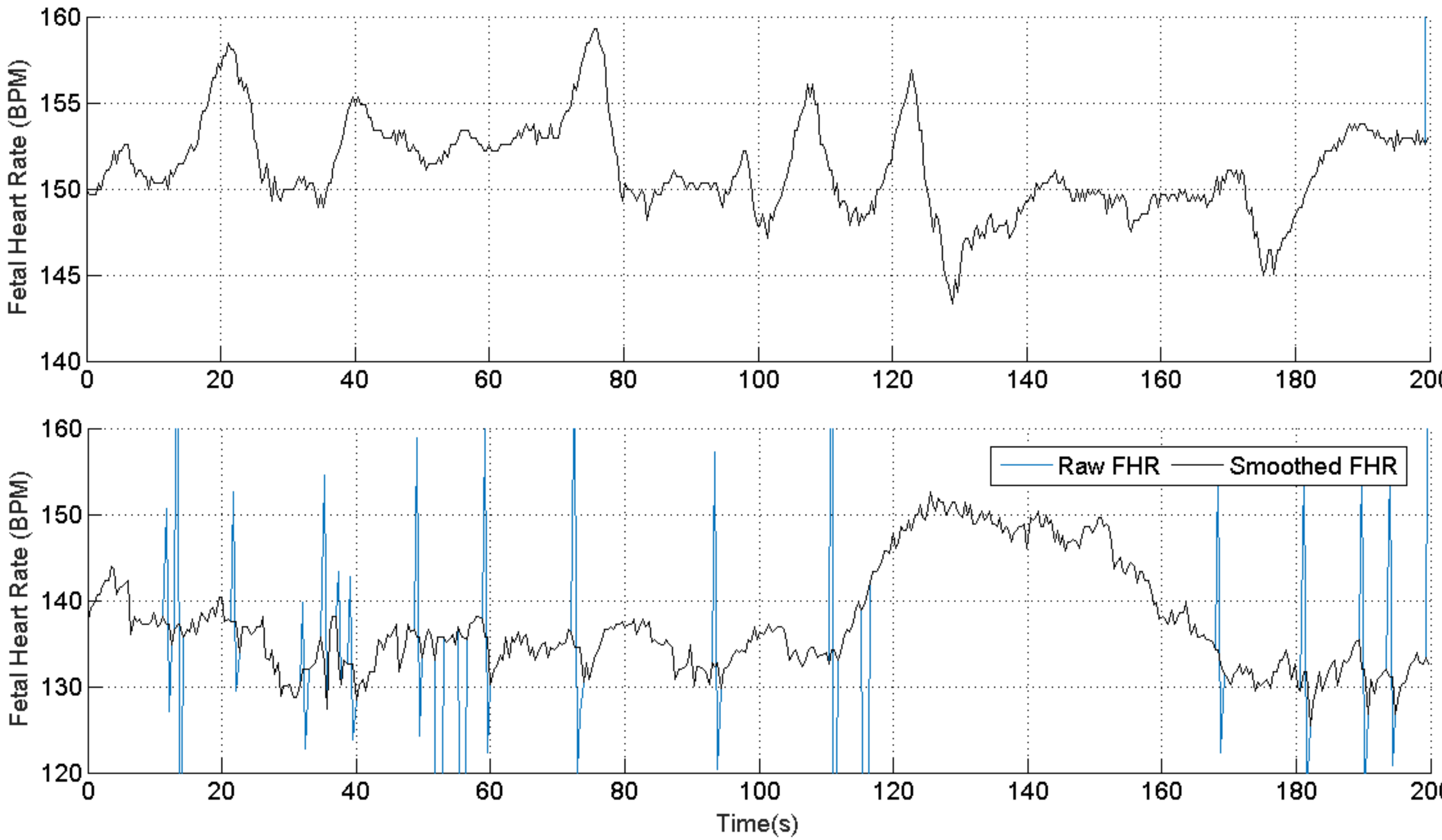
Ref: H. Biglari and R. Sameni, “Fetal motion estimation from noninvasive cardiac signal recordings,” *Physiological Measurement*, vol. 37, no. 11, pp. 2003-2023, November 2016.

Fetal Heart Rate Correction (continued)

Example 2: Twin fetuses (MCG data)

Fetus 1

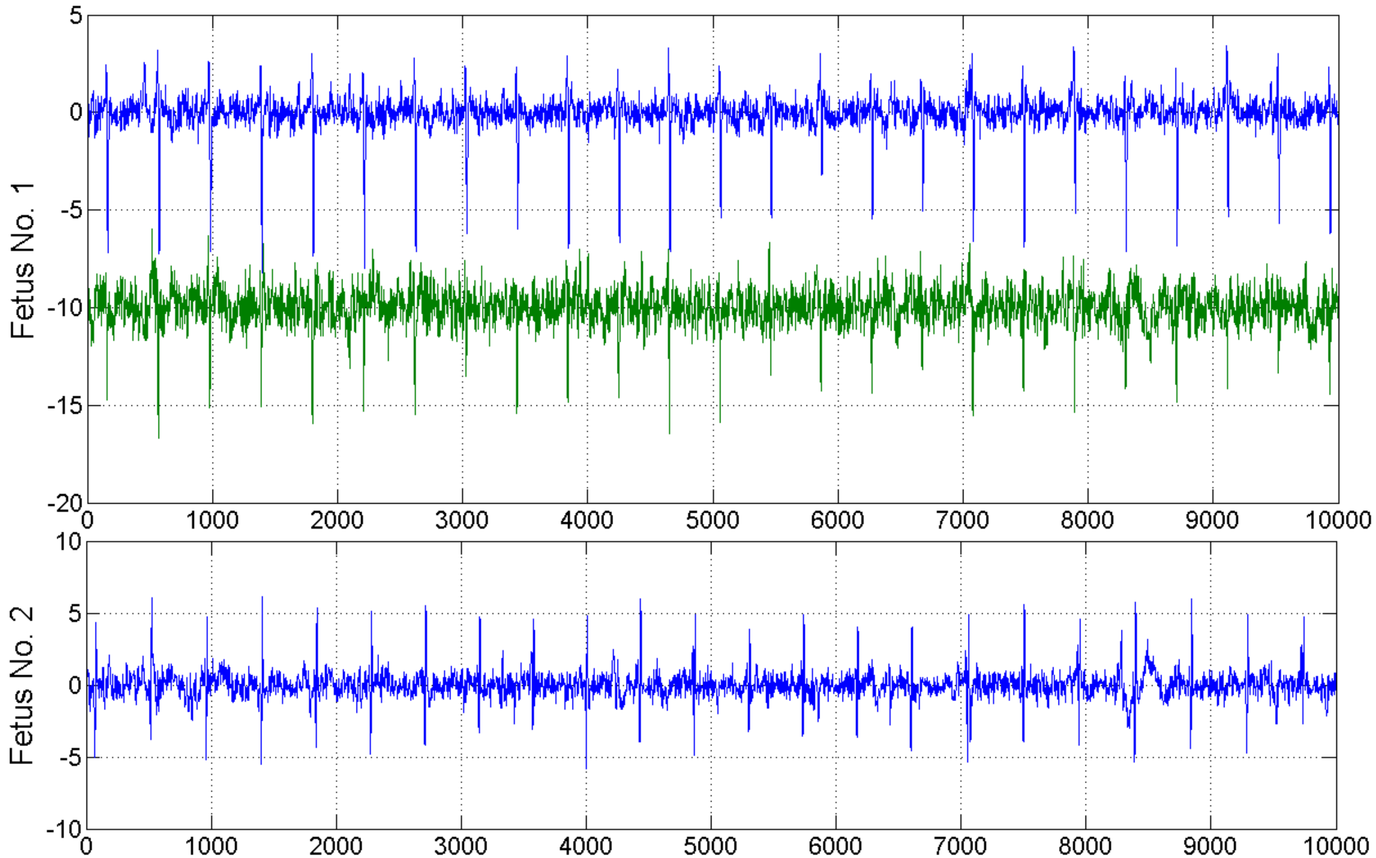
Fetus 2



Ref: H. Biglari and R. Sameni, “Fetal motion estimation from noninvasive cardiac signal recordings,” *Physiological Measurement*, vol. 37, no. 11, pp. 2003-2023, November 2016.

Fetal Heart Rate Correction (continued)

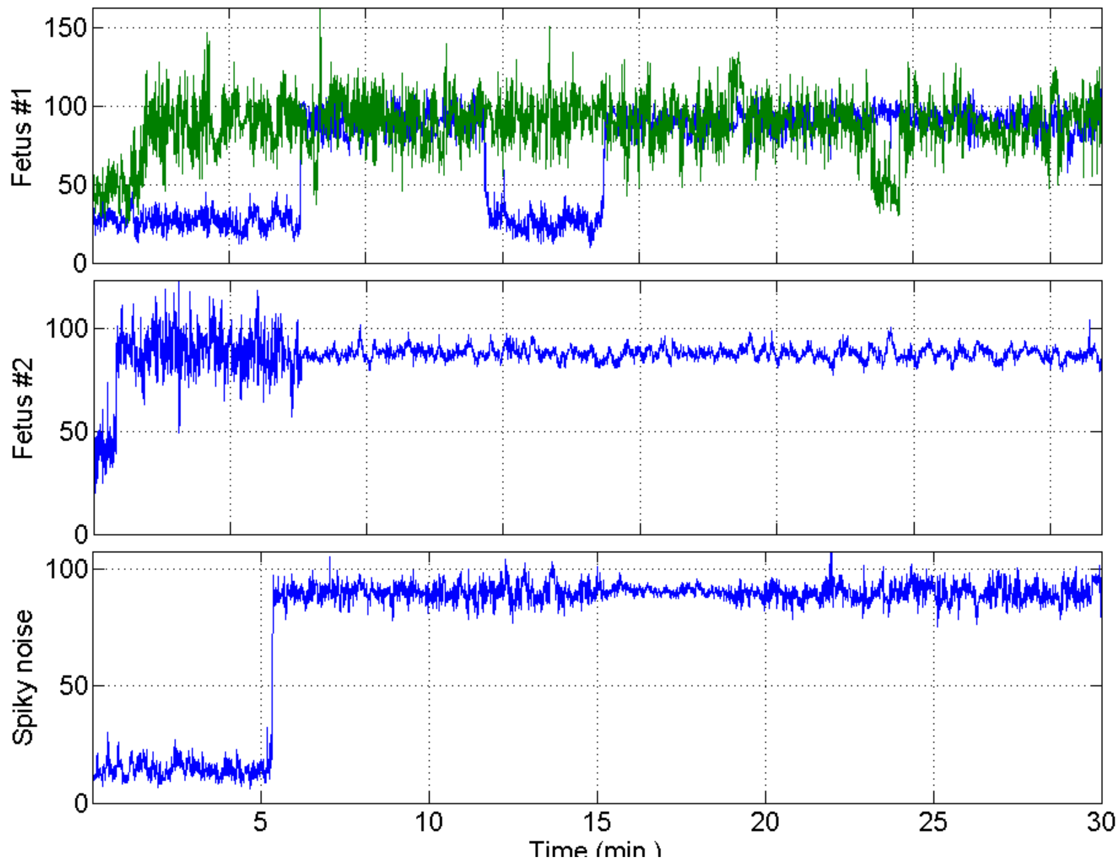
Example 2 (continued): Twin fetuses MCG signals



Ref: Sameni, R. (2008). *Extraction of fetal cardiac signals from an array of maternal abdominal recordings* Doctoral dissertation, Institut National Polytechnique de Grenoble-INPG; Sharif University of Technology.

Fetal Heart Rate Correction (continued)

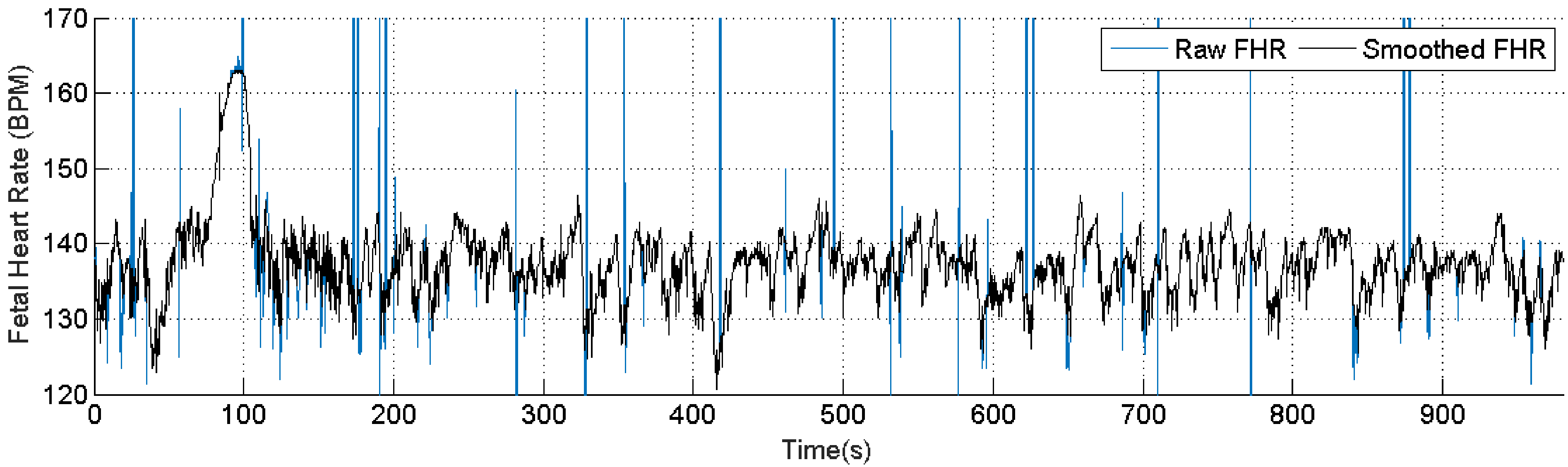
Example 2 (continued): Twin fetuses MCG signals incremental GEVD update



Ref: Sameni, R. (2008). *Extraction of fetal cardiac signals from an array of maternal abdominal recordings* Doctoral dissertation, Institut National Polytechnique de Grenoble-INPG; Sharif University of Technology.

Fetal Heart Rate Correction (continued)

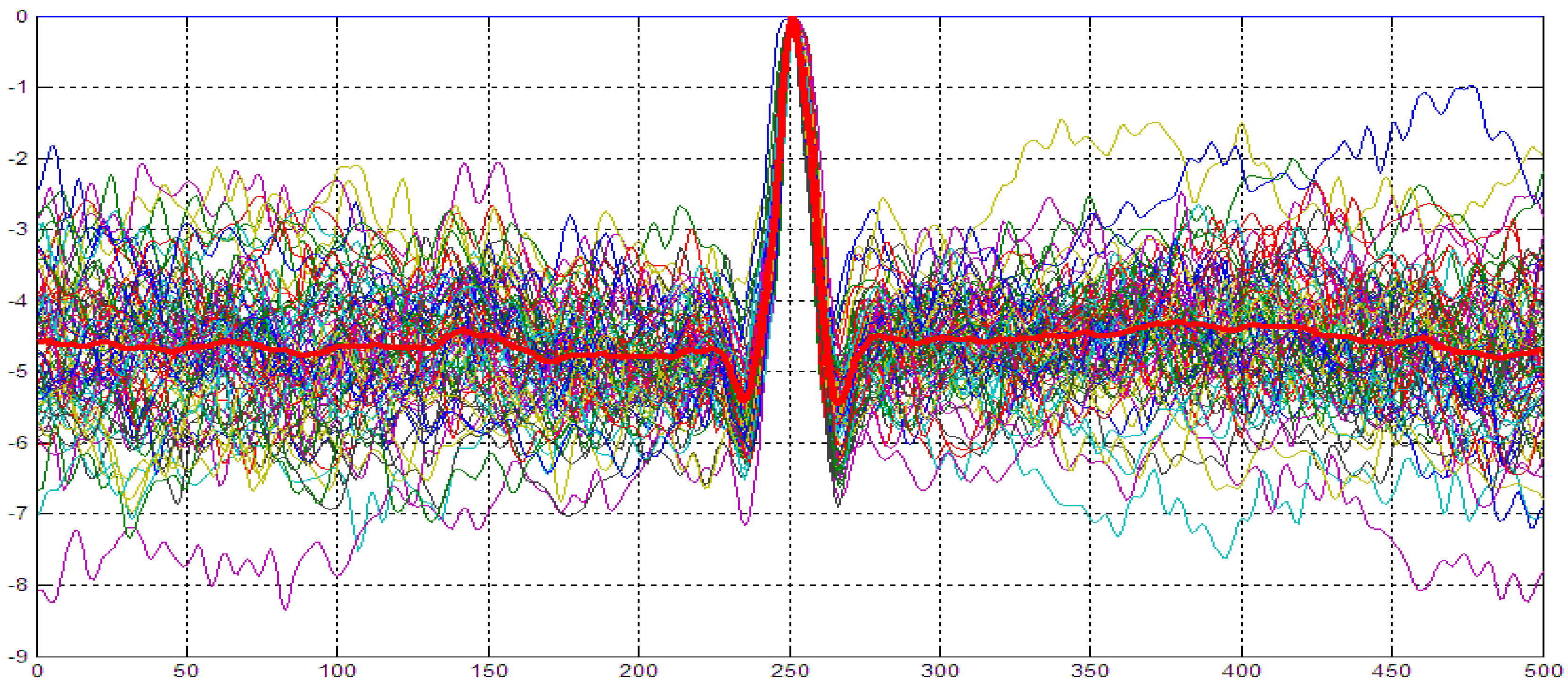
Example 3: Fetal ECG refining



Ref: H. Biglari and R. Sameni, “Fetal motion estimation from noninvasive cardiac signal recordings,” *Physiological Measurement*, vol. 37, no. 11, pp. 2003-2023, November 2016.

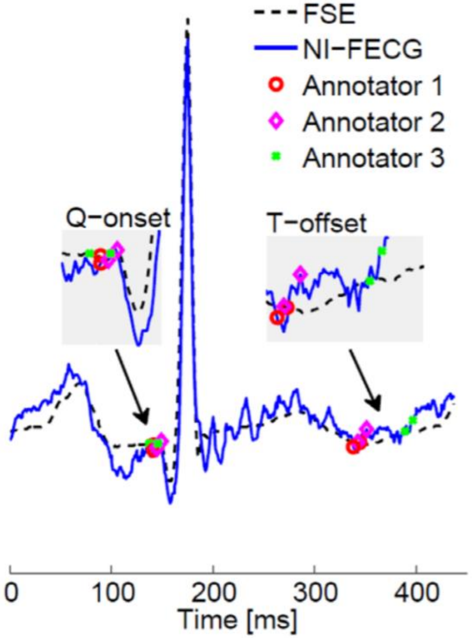
Fetal Morphologic Parameters

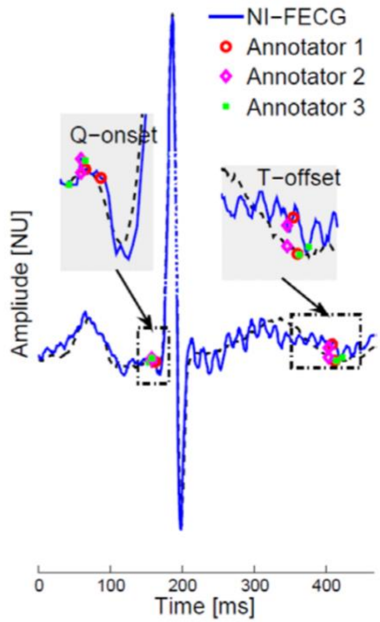
- Average beat morphologies are obtained by beat alignment and (weighted) averaging.
- The beat averaging can either be simple or by normalizing the RR-intervals, using the ECG phase signal $\theta(t)$ introduced before

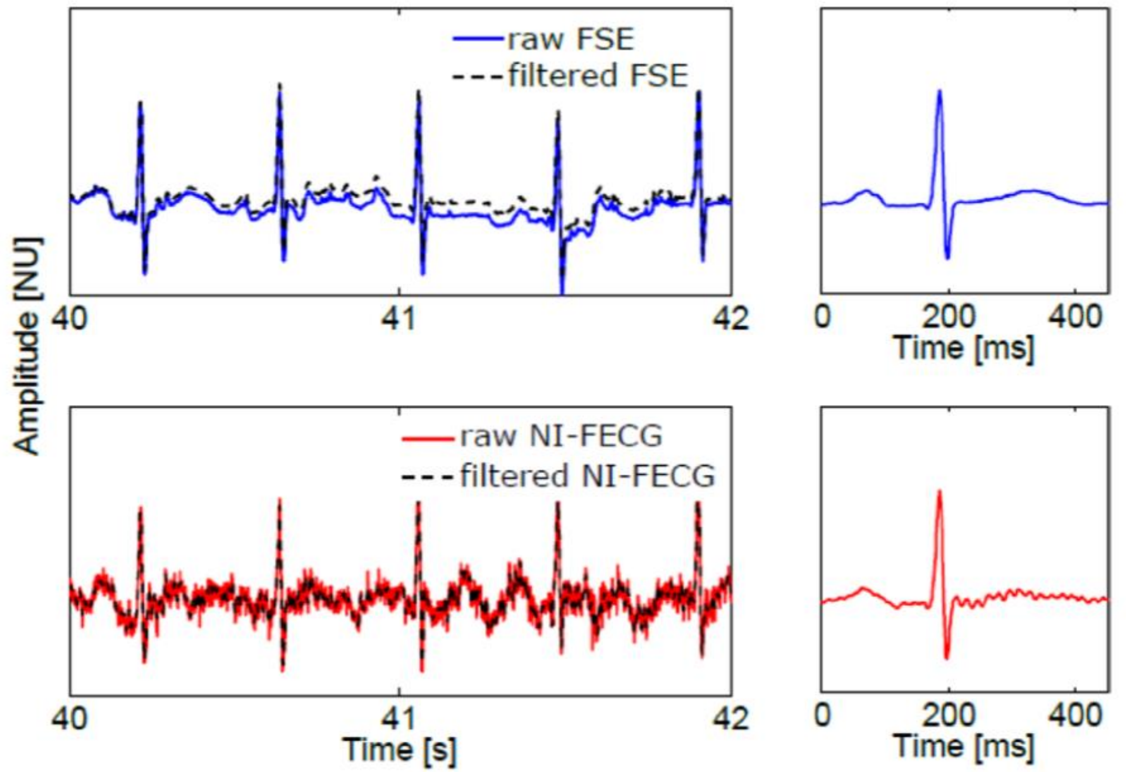


Fetal QT Interval

Example:



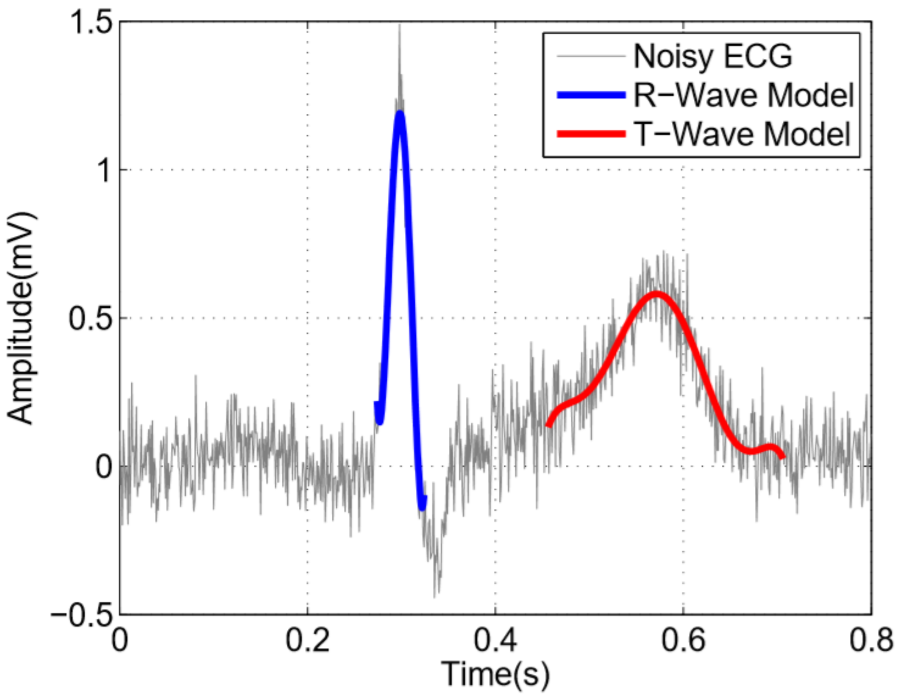


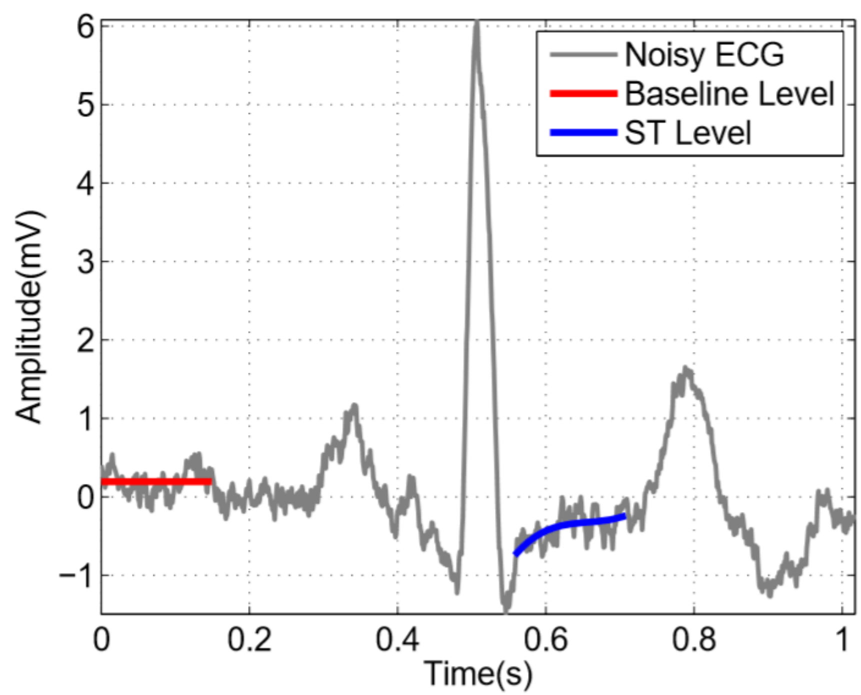


Ref: J. Behar, T. Zhu, J. Oster, A. Niksch, D. Y. Mah, T. Chun, J. Greenberg, C. Tanner, J. Harrop, R. Sameni, J. Ward, A. J. Wolfberg, and G. D. Clifford, “Evaluation of the fetal QT interval using non-invasive fetal ECG technology,” *Physiological Measurement*, vol. 37, no. 9, pp. 1392-1403, September 2016.

Model-Based Parameter Extraction

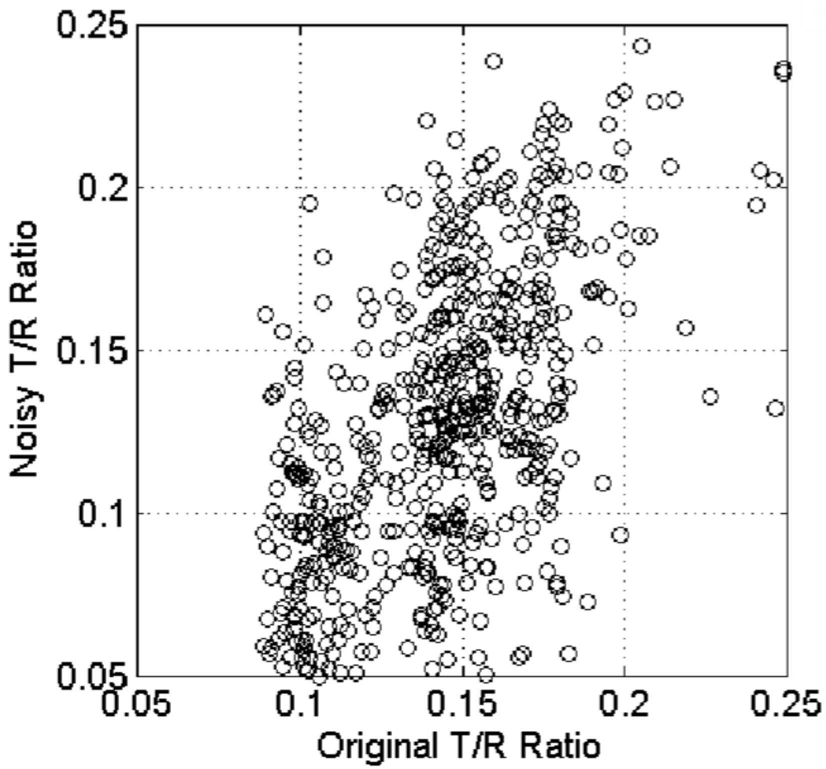
By fitting polynomial (or other) models over ECG segments (or over average beats), more accurate and robust parameters are obtained



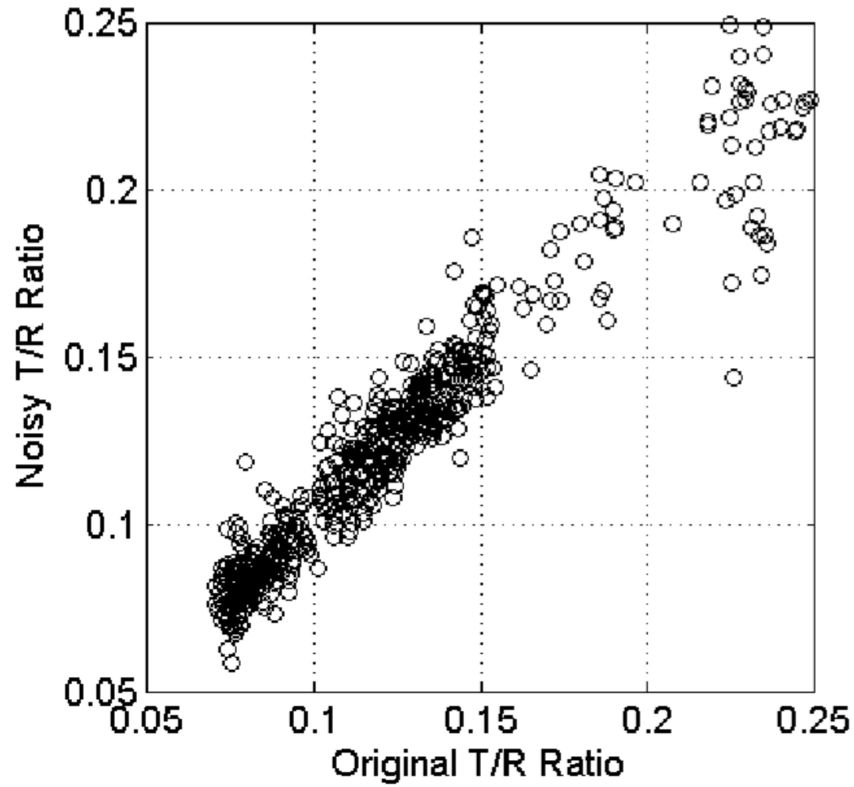


Model-Based Parameter Extraction (continued)

Example 1: T/R ratio calculation with and without model fitting (adult ECG samples)



without model fitting



with model fitting

Advanced Topics

Recent Advances in Fetal Electrocardiography

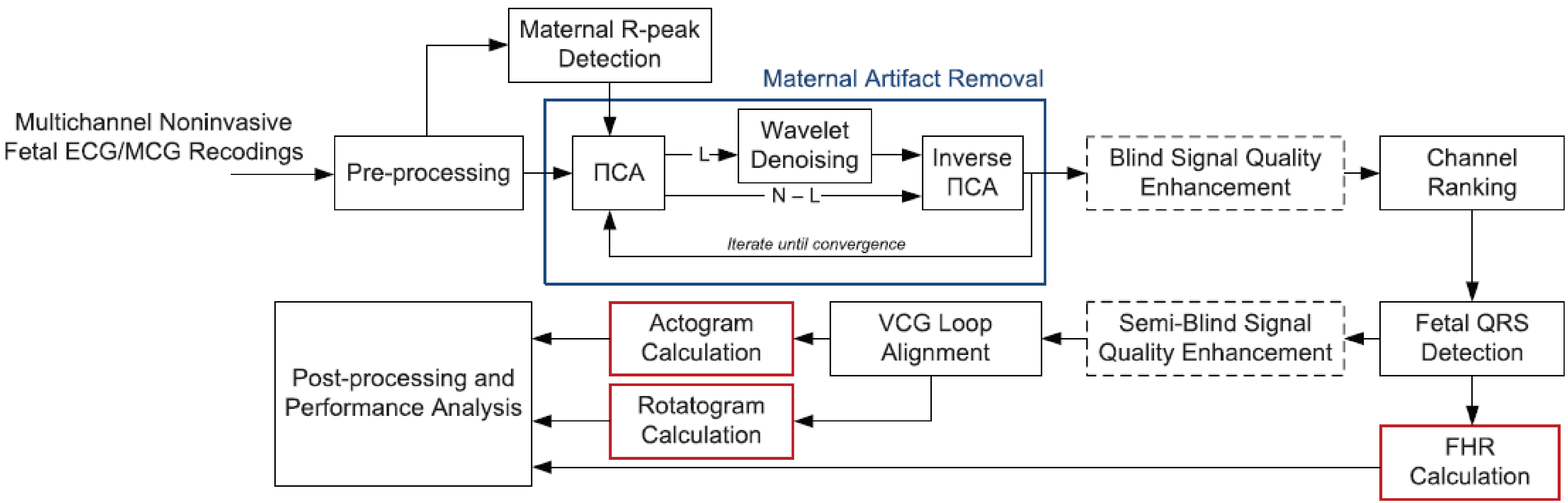
Complementary FECG Inferred Information

- The extracted fetal ECG can be used to extract complementary information regarding the fetal well-being (beyond the fetal heart)
- Example:
  - Fetal motion tracking
  - Fetal position with respect to the maternal body coordinates

Ref: Vullings, R., Peters, C. H. L., Mischi, M., Oei, S. G., & Bergmans, J. W. M. (2008, August). Fetal movement quantification by fetal vectorcardiography: a preliminary study. In *Engineering in Medicine and Biology Society, 2008. EMBS 2008. 30th Annual International Conference of the IEEE* (pp. 1056-1059). IEEE.

Fetal Motion Tracking from fECG

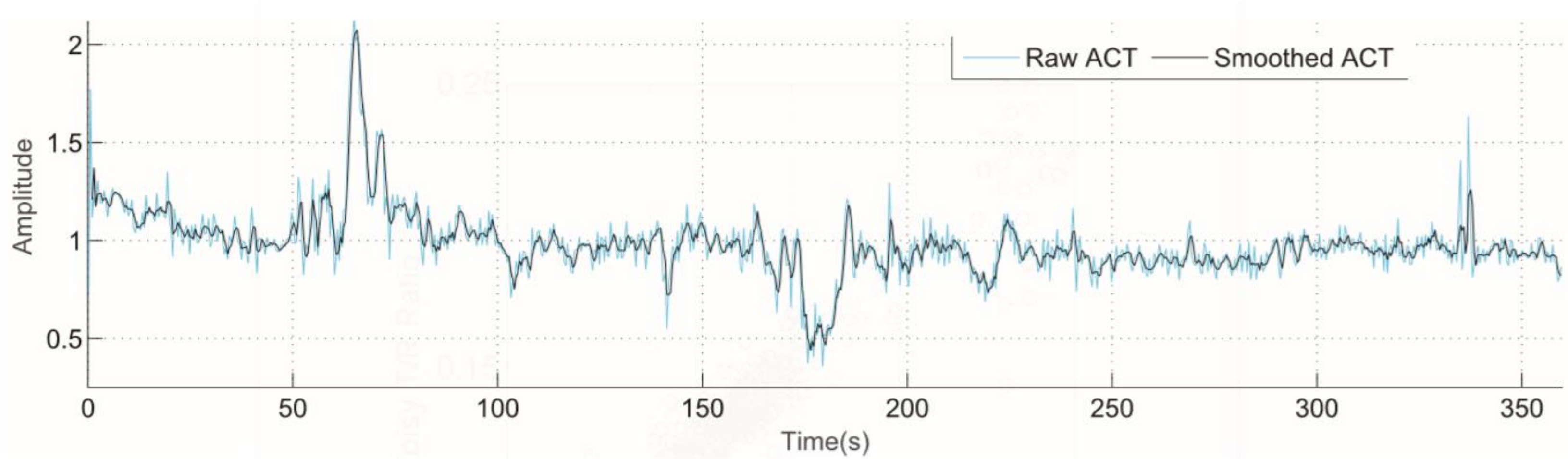
The notion of fetal actogram and rotatogram using fECG beat alignment:

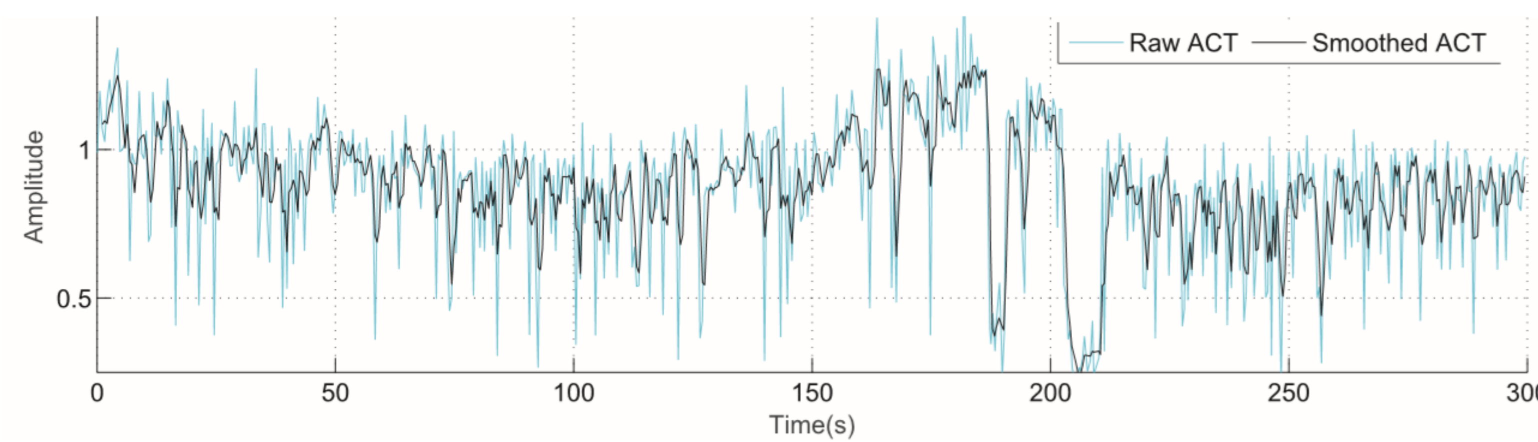


Ref: H. Biglari and R. Sameni, “Fetal motion estimation from noninvasive cardiac signal recordings,” *Physiological Measurement*, vol. 37, no. 11, pp. 2003-2023, November 2016.

Fetal Motion Tracking from fECG (continued)

Example:



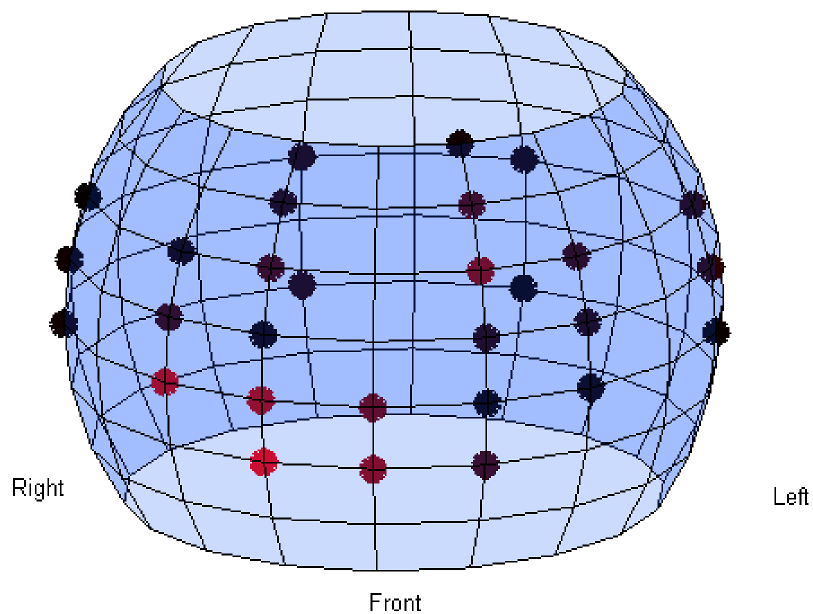


Ref: H. Biglari and R. Sameni, "Fetal motion estimation from noninvasive cardiac signal recordings," *Physiological Measurement*, vol. 37, no. 11, pp. 2003-2023, November 2016.

Fetal Position Extraction

- By mapping the fECG back to the abdominal lead space and calculating the energy distribution of the fECG across the abdominal leads an estimate of the fetal position is obtained
- By mapping the beats onto a "canonical representation" the fetal orientation with respect to the maternal body is obtained.

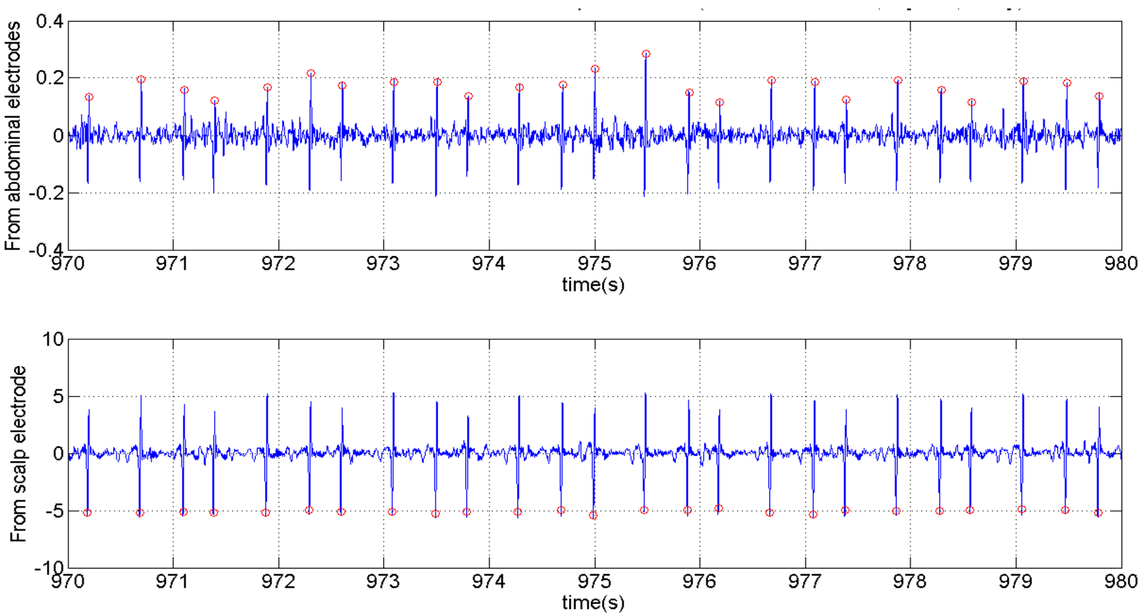
Example:



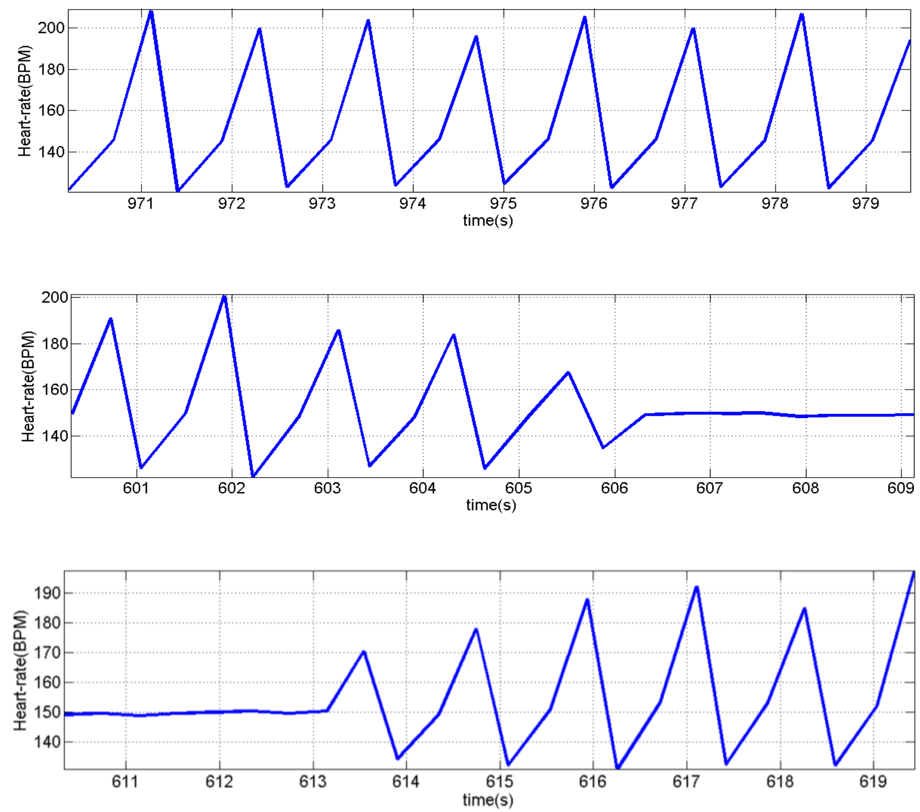
Ref: Clifford, G., Sameni, R., Ward, J., Robinson, J., & Wolfberg, A. J. (2011). Clinically accurate fetal ECG parameters acquired from maternal abdominal sensors. *American Journal of Obstetrics & Gynecology*, *205*(1), 47-e1.

Biphasic and Multiphasic Fetal Heart Rates

Fetal heart rate time-series may demonstrate occasional multiphasic or chaotic behavior



Extracted fECG



Heart-rate segments with multiphasic property

Statistical Performance Bounds on Parameter Estimation

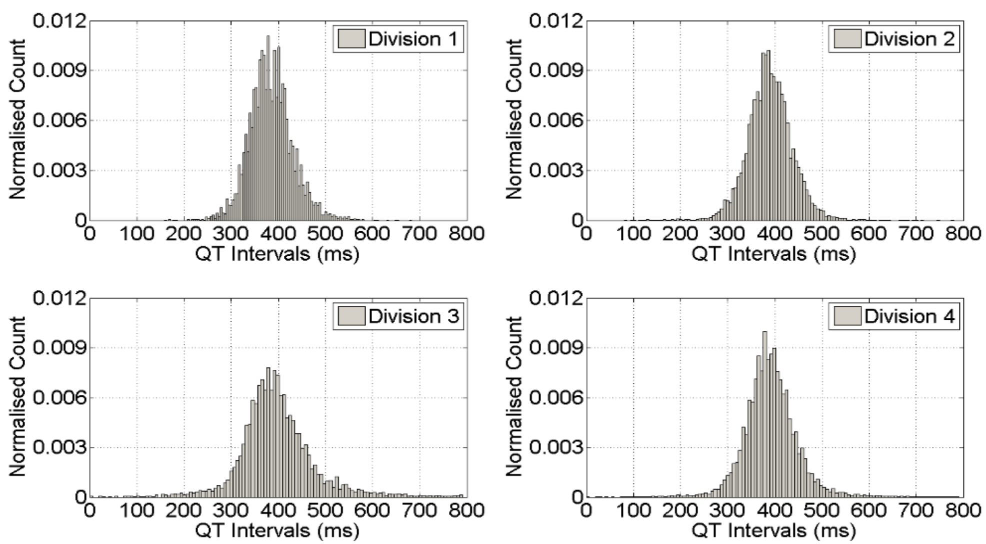
- Recent research have demonstrated that in the context of model-based ECG analysis (either for adults or the fetus), theoretical bounds can be found for the performance of parameter estimators.
- These findings are based on the fact that:
  - ECG parameters have intrinsic beat-wise variability (modeling errors)
  - Measurements and expert annotations are always noisy (observation noise)

Result: A lower limit exists on the achievable parameter estimation error (similar to the well-known Cramér–Rao Lower Bound (CRLB) in estimation theory), which does not depend on the parameter estimation method; but rather relies on the presumed data model.

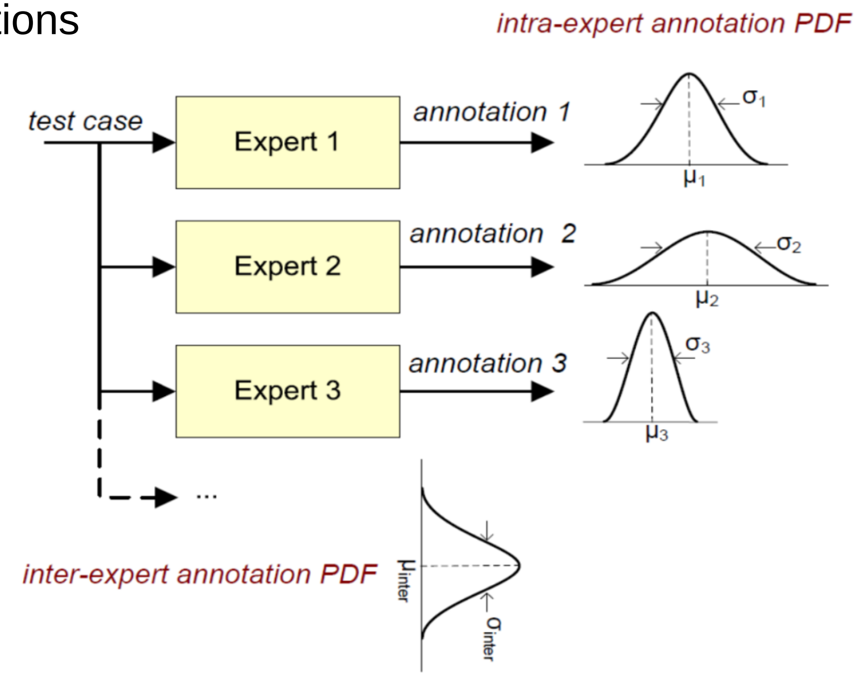
Ref: Current research by D. Fattahi and R. Sameni in Shiraz University

Crowd-Sourced Annotation of Fetal ECG

- A new concept introduced in recent years is that expert annotations (considered as the Gold Standard) have inevitable inter-rate and intra-rater variabilities.
- For an accurate annotation, such variabilities should also be considered in performance assessment
- Recent research has proposed a Bayesian framework for averaging (or obtaining consensus) between a group (crowd) of expert or algorithmic annotations



Ref: Zhu, T., Johnson, A. E., Behar, J., & Clifford, G. D. (2014). Crowd-sourced annotation of ECG signals using contextual information. *Annals of biomedical engineering*, *42*(4), 871-884.



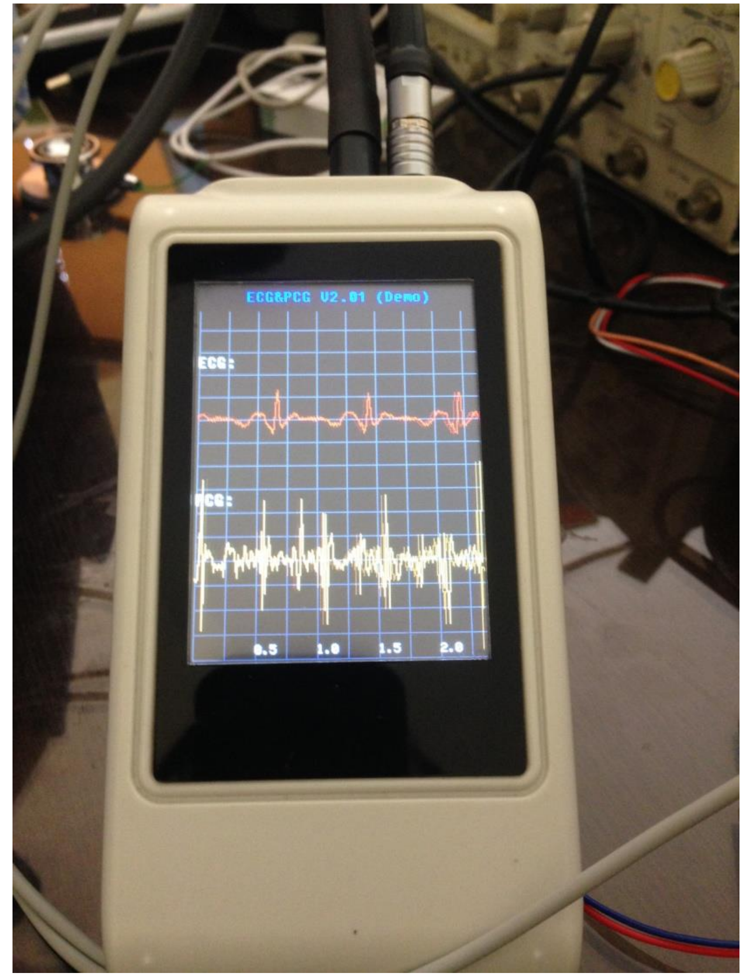
Ref: Current research at Shiraz University (Sameni© 2018)

Multimodal ECG-PCG Analysis

- A rather new trend of research is to use a combination of different and simultaneously acquired modalities for adult/fetal cardiac monitoring
- The ultimate objective is to study the electro-mechanical coupling of the heart

Example: simultaneous adult /fetal ECG and PCG

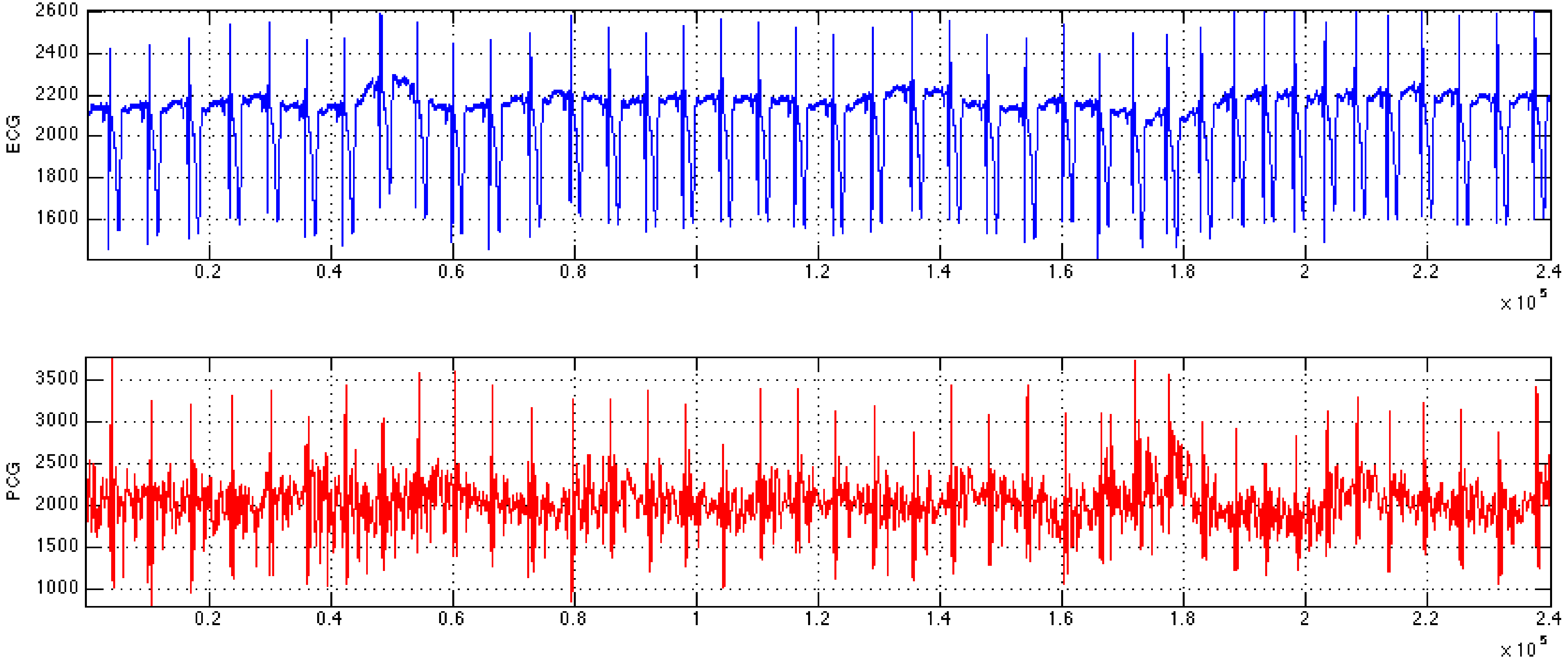
Ref:
•Samieinasab, M., & Sameni, R. (2015, May). Fetal phonocardiogram extraction using single channel blind source separation. In Electrical Engineering (ICEE), 2015 23rd Iranian Conference on (pp. 78-83). IEEE.
•Shiraz University Fetal Heart Sounds Database (SUFHSDB), Physionet. Available Online: https://physionet.org/physiobank/database/sufhsdb/



Current research at Shiraz University (Sameni© 2018)

Multimodal ECG-PCG Analysis (continued)

Example: Simultaneous ECG and PCG



Further Reading

Additional References

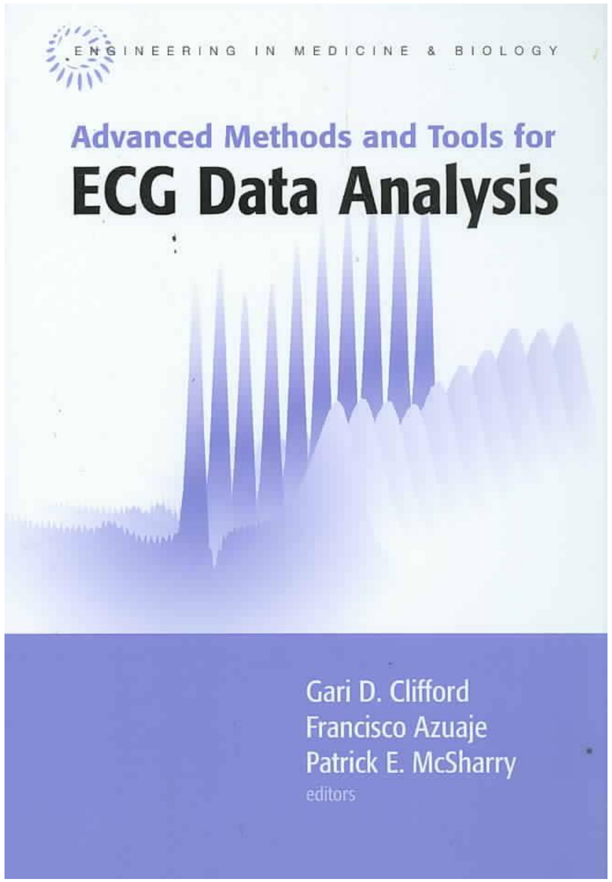
Source Codes and Algorithms to Consider

- Source codes for this workshop: The Open-Source Electrophysiological Toolbox (OSET), Version 3.14 (the π version), www.oset.ir; a public repository

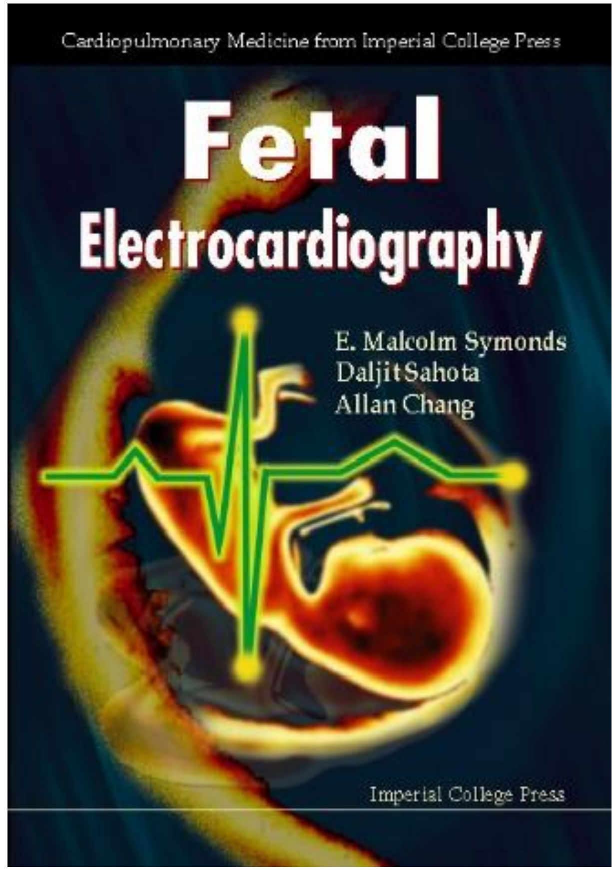
- Blind source separation:
  - FastICA. Online available: http://research.ics.aalto.fi/ica/fastica/
  - Joint Approximation Diagonalization of Eigen-matrices (JADE)
  - Second order blind identification (SOBI)

- Public fetal ECG databases:
  - OSET fetal ECG database: www.oset.ir
  - Non-Invasive Fetal Electrocardiogram Database: https://physionet.org/pn3/nifecgdb/
  - Abdominal and Direct Fetal Electrocardiogram Database: https://www.physionet.org/physiobank/database/adfecgdb/

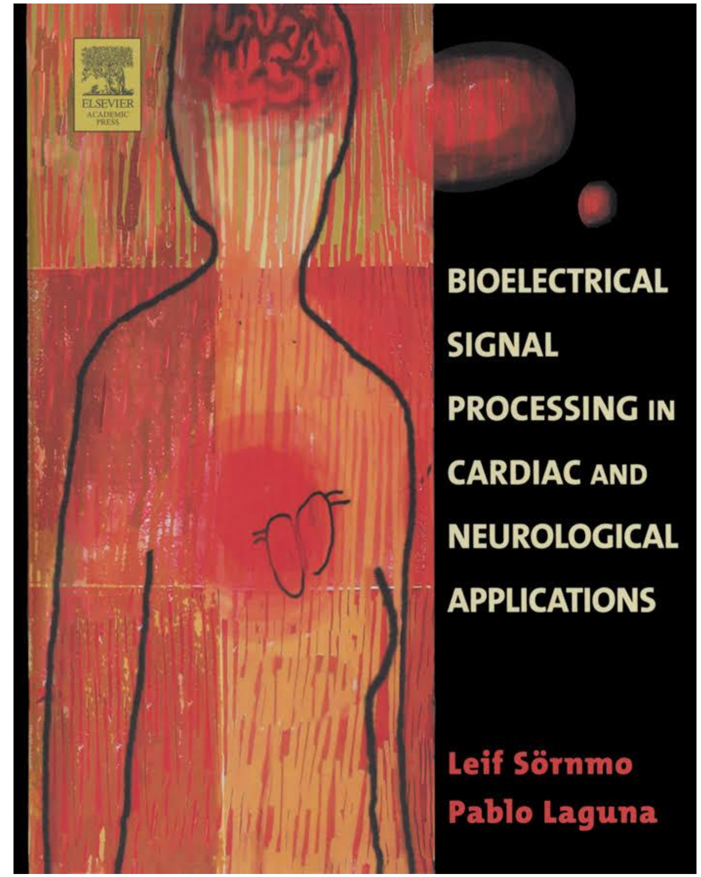
Textbooks on ECG Signal Processing



G. Clifford, F. Azuaje, and P. McSharry, *Advanced methods and tools for ECG data analysis*, ser. Artech House engineering in medicine & biology series. Artech House, 2006.



E. M. Symonds, D. Sahota, and A. Chang, *Fetal Electrocardiography*, World Scientific, 2001



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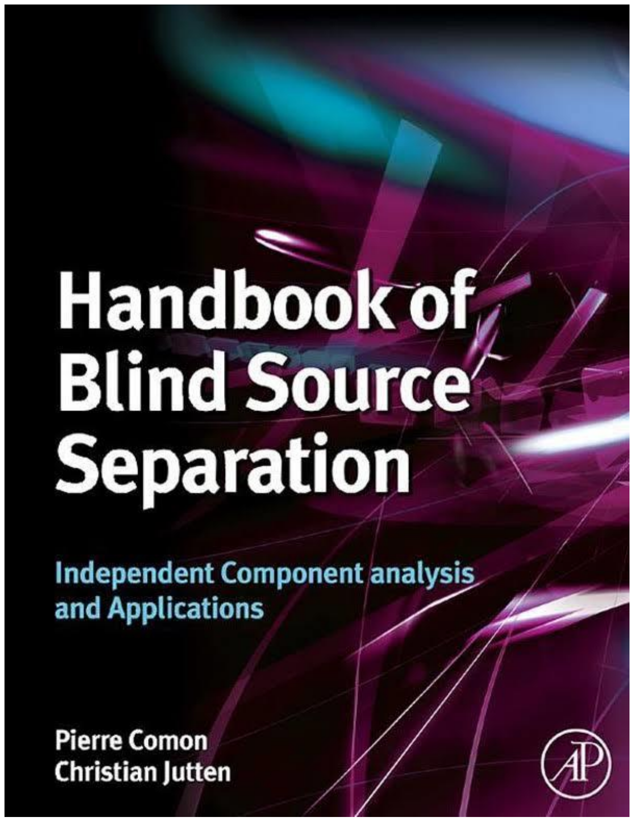
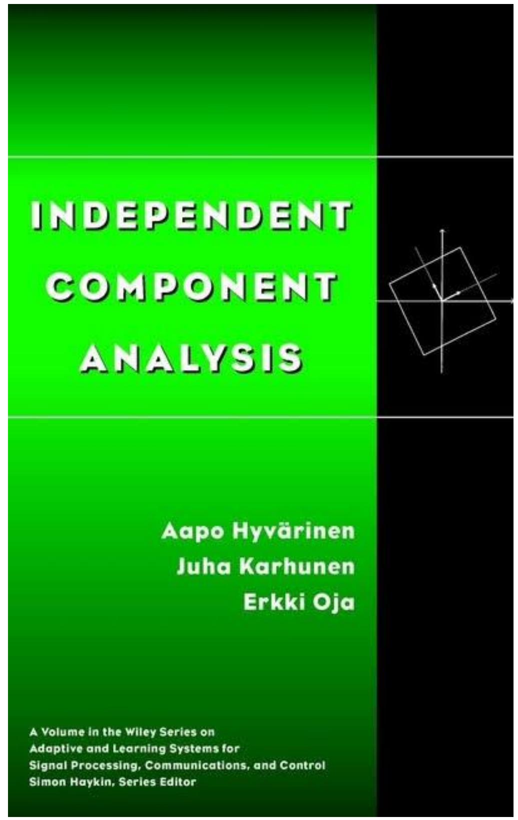
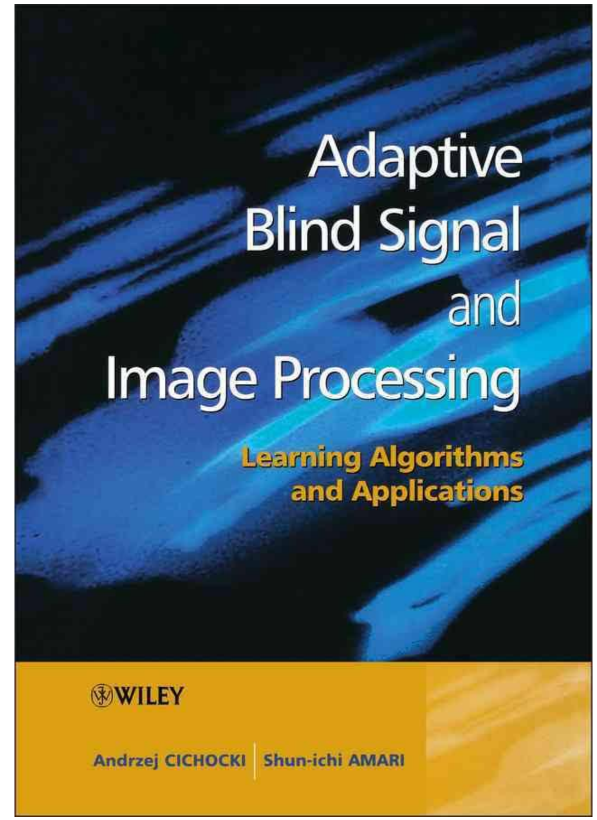
Textbooks on BSS and ICA



Comon, P., & Jutten, C. (Eds.). (2010). *Handbook of Blind Source Separation: Independent component analysis and applications*. Academic press.

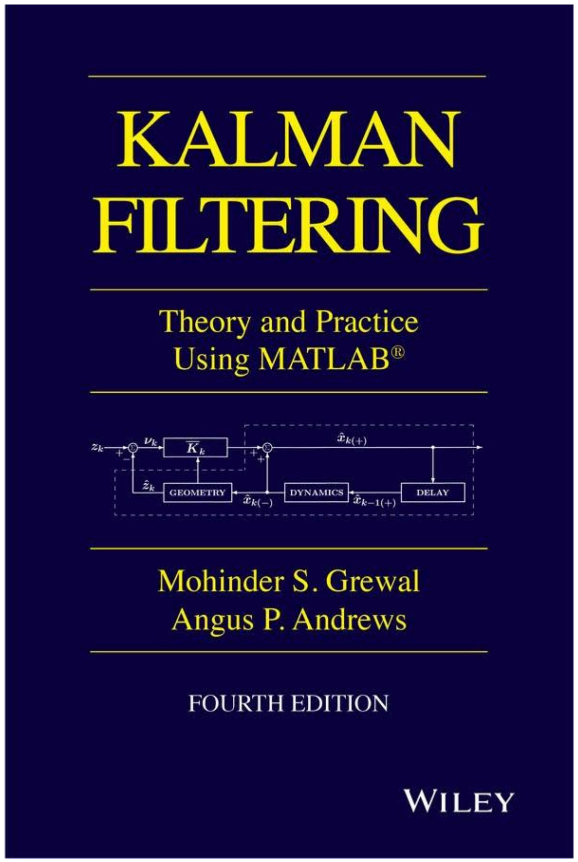


Hyvärinen, A., Karhunen, J., & Oja, E. (2004). *Independent component analysis*, John Wiley & Sons.

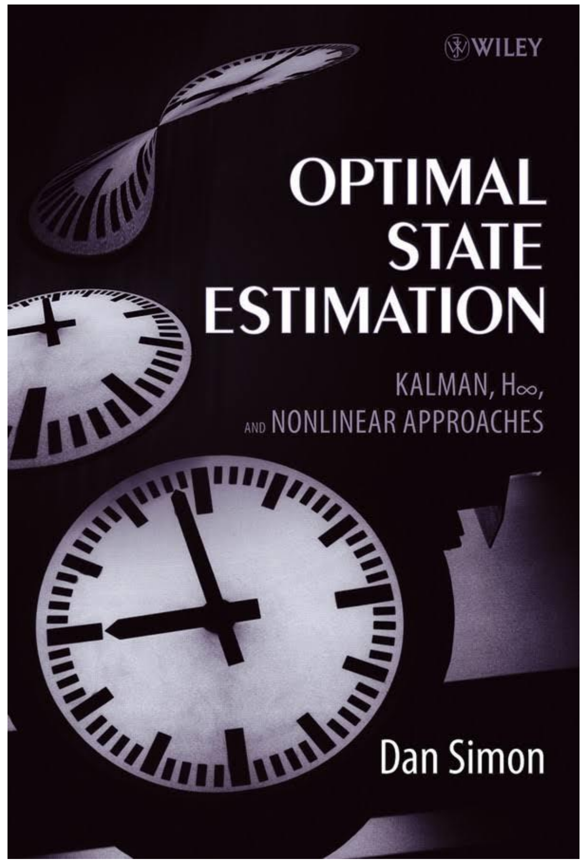


Cichocki, A., & Amari, S. I. (2002). *Adaptive blind signal and image processing: learning algorithms and applications* (Vol. 1). John Wiley & Sons.

Textbooks on Applied Estimation Theory and Kalman Filtering



M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice Using Matlab*, 4th ed. John Wiley & Sons, Inc., 2015.



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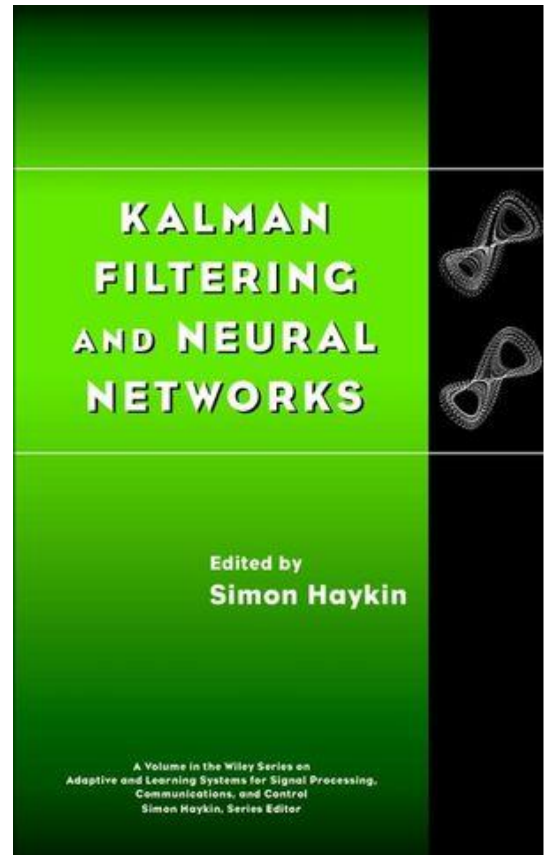

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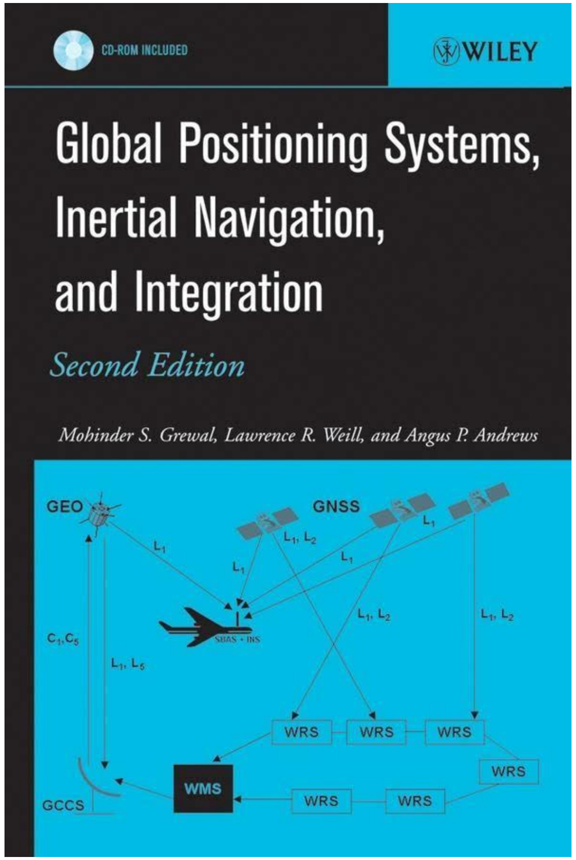
Textbooks on Applied Estimation Theory and Kalman Filtering (continued)



M. Grewal, L. Weill, and A. Andrews, *Global Positioning Systems, Inertial Navigation, and Integration*. Wiley, 2007

Refer to Chapter 8 for some nice Kalman filter engineering techniques

Introduction to the Matlab Laboratory Sessions

A Few Words on Open-Source Biomedical Signal Processing Projects

Why to contribute in open-source projects?

- To present a standard implementation of your algorithms: other researchers can repeat your results in your way (not just based on their understanding of your articles)
- Many people will read, debug and cite your work for free!
- You remember what you did and remain committed to the topic (your ideas are not just forgotten after graduation)
- You get invited to summer schools and find new friends…☺

Caution: Be aware of what you release, if you have a work-for-hire contract or intend to patent your ideas, consult your supervisor or manager before any public release

Version Control Software and Cloud Source Repositories

Why to use version control software?

- Higher reliability: As Master's and PhD students, data and source codes are your most precious assets. I've seen many students who lost several months of their research after a fatal OS or hardware crash of their PC. It actually just happened for me before this talk!... ☺
- Having a Master Version: In team projects or when working on a single source-code from different computers, you commonly end-up with multiple copies of the same source codes, with minor "forgotten" differences, which make your codes vulnerable
- Recent versions of Matlab and many other popular signal processing software support version control
- Use your favorite source control software (Git, SVN, etc.) and repository structure (preferably placed over a cloud or on distanced PC/Data-Centers)

Caution: Don't overuse cloud repositories; just in case you need to go to class when the network is down ☺

The Open-Source Electrophysiological Toolbox (OSET) – www.oset.ir

- OSET is a collection of electrophysiological data and open source codes for biological signal generation, modeling, processing, and filtering, originally released in 2006.
- It is distributed under the GNU General Public License and may be freely used or modified under the specified terms of use.
- The source codes have been mainly developed in Matlab and partially in C++; but contributions in other languages are welcome.
- Starting from Version 3.14 (the π version), OSET is accessible and will be updated on a Git repository.

How to download and use OSET?

- Installation: you can either
  1. Download the entire project in zip format from: https://gitlab.com/rsameni/OSET.git, or
  2. Clone the repository on your personal computers
- Add the repository and its sub-folders to your Matlab path
- Start using it!

Note: remember to check for updates or by simply fetching (using the Git pull command) the latest updates regularly

Matlab Laboratory Highlights

- In this Matlab laboratory, we study and practice some of the basic tools of OSET for adult/fetal ECG signal processing, including:
  1. Necessity of signal visualization
  2. Signal modeling
  3. Preprocessing (BP filtering, wavelet denoising, notch-filtering)
  4. Single-channel fECG extraction
  5. Multichannel fECG extraction and processing
  6. Post-processing the extracted fECG

Matlab Laboratory Highlights (continued)

Quick notes to remember from the lab

Coding style:

- Writing standard codes that are simply adaptable to different sampling frequencies (use ratios of any desired frequency divided by fs instead of of using fixed numbers)
- Separating data from functions
- Separating general purpose functions from demos and test files

Visualization: Why and how to visualize fetal ECG recordings

Preprocessing:

- Filter design technique (Matlab FDATool)
- testECGSpectrum.m (for visualization)
- testFilters.m (for baseline wander removal)
- testfECGPreprocessing.m
- testBaselineWanderRemovalMedianFilter.m
- testANC.m (for poweline)
- testECGAdaptiveFilter.m
- testECGAdaptiveFilterMultipleDelay.m
- testECGKFNotch.m

Matlab Laboratory Highlights (continued)

Quick notes to remember from the lab

R-peak detection:
- testPeakDetection.m
- testMFPeakDetector.m
- testFetalHRSeries.m

mECG cancellation:
- testMECGTemplateSubtraction.m
- testMeanECGExtraction.m
- testECGKalmanFilter.m
- testMECGKalmanFilterRemoval.m

Single-channel fECG extraction from noise (after mECG cancellation):
- testFetalECGExtraction.m
- testFECGKalmanFilter1.m, testFECGKalmanFilter2.m

Multichannel fECG extraction:
- Necessity of using linear-phase filters or zero-phase filters (by forward-backward filtering using filtfilt) for multichannel data to preserve the sample-wise synchrony
- A short tutorial on Matlab FDATool

Matlab Laboratory Highlights (continued)

Quick notes to remember from the lab

Multichannel fECG extraction (continued):

- Be cautious about dimension reduction by PCA: the fECG has low contribution in the signal energy and may be eliminated during PCA dimension reduction.
- testPCAICAPiCAfECGExtraction.m
- testICAPiCAAfterMECGCancellation.m (using the deflation algorithm)
- testPCAICAPiCAfECGDenoising.m (denoising using BSS and semi-BSS methods)

Curse-of-dimensionality in using ICA for high-dimensional data:

- testSpikesBumpsRealData.m
- testSpikesBumpsSimulatedData.m

Tools for generating synthetic and realistic single/multi-channel adult and fetal ECG mixtures:

- testSyntheticMaternalFetalECGGeneration1.m
- testSyntheticMaternalFetalECGGeneration2.m